THE DIMENSIONS OF INDIVIDUAL POINTS IN EUCLIDEAN SPACE

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The recent theory of constructive dimension uses the theory of computing to assign a dimension to every *individual point* in Euclidean space. These dimensions appear to be geometrically meaningful. For example, we now know the following.

1. The *classical* Hausdorff dimension of any set X that is a union of computably closed sets is simply the supremum of the dimensions of the individual points $x \in X$. (work with Hitchcock)
2. Every point on any computable curve of finite length has dimension at most 1 (but not conversely). (work with Gu and Mayordomo)
3. For any point x in any computably self-similar fractal F and any sequence T that canonically codes the location of $x \in F$, the dimension of x is given by $\dim(x) = \dim(F) \dim^{\pi}(T)$, where $\dim(F)$ is the similarity dimension of F and $\dim^{\pi}(T)$ is the dimension of the sequence T with respect to a natural probability measure π induced by F. (work with Mayordomo)

This talk will survey these developments and suggest directions for future research into how the dimensions of points and related ideas from the theory of computing interact with geometric measure theory.