ComS 573: Machine Learning
Spring 2012

Homework 1
Due Friday, January 20, 2012 in class

Note: Please do not hesitate to contact the instructor or TA if you have difficulty understanding or getting started with solving any of the problems.

1. (20 pts.) Suppose that two variables X and Z are statistically independent. Show that

$$E[X + Z] = E[X] + E[Z]$$

$$var[X + Z] = var[X] + var[Z]$$

$$cov(X, Z) = 0$$

2. (20 pts.) This exercise investigates the way in which conditional independence relationships affect the amount of information needed for probabilistic calculations.

   (a) Suppose we wish to calculate $P(h|e_1, e_2)$ and we have no conditional independence information. Which of the following sets of numbers are sufficient for the calculation?

      i. $P(E_1, E_2), P(H), P(E_1|H), P(E_2|H)$
      ii. $P(E_1, E_2), P(H), P(E_1, E_2|H)$
      iii. $P(H), P(E_1|H), P(E_2|H)$

   (b) Suppose we know that $P(E_1|H, E_2) = P(E_1|H)$ for all values of H, E_1, E_2. Now which of the three sets are sufficient.

3. (10 pts.) Suppose you are a witness to a nighttime hit-and-run accident involving a taxi in Athens. All taxis in Athens are blue or green. You swear, under oath, that the taxi was blue. Extensive testing shows that, under the dim lighting conditions, discrimination between blue and greeen is 75% reliable. Is it possible to calculate the most likely color for the taxi? (Hint: distinguish carefully between the proposition that the taxi is blue and the proposition that it appears blue.)
   What about now, given that 9 out of 10 Athenian taxis are green?

4. (20 pts.) Suppose you are given a bag containing n unbiased coins. You are told that $n - 1$ of these coins are normal, with heads on one side and tails on the other, whereas one coin is a fake, with heads on both sides.

   (a) Suppose you reach into the bag, pick out a coin uniformly at random, flip it, and get a head. What is the (conditional) probability that the coin you chose is the fake coin?

   (b) Suppose you flip the coin you chose for a total of k times and see k heads. Now what is the (conditional) probability that the coin you chose is the fake coin?

(c) Suppose you wanted to decide whether the chosen coin was fake by flipping it k times. The decision procedure retures FAKE if all k flips come up heads, otherwise it returns NORMAL. What is the (unconditional) probability that this procedure makes an error?

5. (10 pts.) Three prisoners, A, B, and C, are locked in their cells. It is common knowledge that one of them will be executed the next day and the others pardoned. Only the governor knows which one will be executed. Prisoner A asks the guard a favor: "Please ask the governor who will be executed, and then take a message to one of my friends B or C to let him know that he will be pardoned in the morning." The guard agrees, and comes back later and tells A that he gave the pardon message to B.
What are A's chances of being executed, given this information? (Answer this *mathematically*.)