

ComS 573: Machine Learning

Spring 2008

Homework 1

Due Friday, January 25, 2008, in class

Note: Please do not hesitate to contact the instructor or TA if you have difficulty understanding or getting started with solving any of the problems.

1. (20 pts.) It is often useful to consider the effect of some propositions in the context of some general background evidence $E = e$, rather than in the complete absence of information.

(a) Prove the general product rule:

$$P(X, Y|e) = P(X|Y, e)P(Y|e)$$

(b) Prove the general Bayes' rule

$$P(Y|X, e) = \frac{P(X|Y, e)P(Y|e)}{P(X|e)}$$

2. (20 pts.) Suppose that two variables X and Z are statistically independent. Show that

$$E[X + Z] = E[X] + E[Z]$$

$$\text{var}[X + Z] = \text{var}[X] + \text{var}[Z]$$

$$\text{cov}(X, Z) = 0$$

3. (20 pts.) This exercise investigates the way in which conditional independence relationships affect the amount of information needed for probabilistic calculations.

(a) Suppose we wish to calculate $P(h|e_1, e_2)$ and we have no conditional independence information. Which of the following sets of numbers are sufficient for the calculation?

i. $P(E_1, E_2), P(H), P(E_1|H), P(E_2|H)$

ii. $P(E_1, E_2), P(H), P(E_1, E_2|H)$

iii. $P(H), P(E_1|H), P(E_2|H)$

(b) Suppose we know that $P(E_1|H, E_2) = P(E_1|H)$ for all values of H, E_1, E_2 . Now which of the three sets are sufficient.

4. (20 pts.) Suppose you are a witness to a nighttime hit-and-run accident involving a taxi in Athens. All taxis in Athens are blue or green. You swear, under oath, that the taxi was blue. Extensive testing shows that, under the dim lighting conditions, discrimination between blue and green is 75% reliable. Is it possible to calculate the most likely color for the taxi? (Hint: distinguish carefully between the proposition that the taxi is blue and the proposition that it appears blue.)

What about now, given that 9 out of 10 Athenian taxis are green?

5. (20 pts.) Suppose you are given a bag containing n unbiased coins. You are told that $n - 1$ of these coins are normal, with heads on one side and tails on the other, whereas one coin is a fake, with heads on both sides.
- (a) Suppose you reach into the bag, pick out a coin uniformly at random, flip it, and get a head. What is the (conditional) probability that the coin you chose is the fake coin?
 - (b) Suppose you flip the coin you chose for a total of k times and see k heads. Now what is the (conditional) probability that the coin you chose is the fake coin?
 - (c) Suppose you wanted to decide whether the chosen coin was fake by flipping it k times. The decision procedure returns FAKE if all k flips come up heads, otherwise it returns NORMAL. What is the (unconditional) probability that this procedure makes an error?