

## HOMEWORK 4 - MATH 325

DUE DATE: After Chapter 6 has been covered!

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Read each problem very carefully before starting to solve it. A few randomly selected problems will be graded for a total of 10 points. It is necessary to show your work.

GOOD LUCK!!

1. Use Figure 5.24 on page 75 of your textbook to show how to construct a pair of lengths  $x, y$  with given difference  $y - x = a$  and given product  $xy = b^2$ . Make sure to prove that your construction works!
2. Use Figure 5.11 of your textbook to prove that for any lengths  $a$  and  $b$ ,  $\sqrt{ab} \leq \frac{1}{2}(a + b)$ . This fact is sometimes called the *algebraic-geometric mean inequality*.
3. Let  $\overline{AB}$  and  $\overline{CD}$  be parallel chords in a circle. Prove that  $\widehat{CAB} \cong \widehat{DBA}$ .
4. Given a circle with center  $O$  and three tangent lines  $l_1, l_2$  and  $l_3$  such that  $l_1$  and  $l_2$  are parallel and  $l_3$  intersects  $l_1$  at  $A$  and  $l_2$  at  $B$ , prove that  $\widehat{AOB} = 90^\circ$ .
5. Given a line segment  $\overline{AB}$ , define  $S$  to be the set of points  $C$  on a given side of  $\overline{AB}$  such that  $\widehat{ACB} = 45^\circ$ . What can you say about  $S$ ?
6. For which  $n$  satisfying  $3 \leq n \leq 25$  is it possible to construct a regular  $n$ -gon?
7. For which of these  $n$  is a regular  $n$ -gon constructible?  
(a) 144   (b) 85   (c) 176   (d) 100   (e) 128.
8. What lower bounds do you get for  $C$  and  $A$  if you use a square inscribed in the circle?
9. What upper bounds do you get for  $C$  and  $A$  if you use a hexagon circumscribed about the circle?