Com S 477/577 Problem Solving Techniques for Applied Computer Science

Exam 1

2:10-3:25am
Thursday, Oct 16, 2008

Name: ________________________________

ID (4-digit): ___ ___ ___ ___

1	2	3	4	5	Total
44	14	16	12	14	100

1. [**44 pts**] *Short Questions*

(a) [**12 pts**] Determine if the following statements are true or false. For each statement, mark only the answer you think is correct.

(i) On a sphere of radius r there exists a curve that has curvature less than $\frac{1}{r}$ at some point.

true ______ false ______

(ii) The two principal vectors at a surface point are always orthogonal to each other.

true ______ false ______

(iii) Two surface patches $\boldsymbol{\sigma}_1, \boldsymbol{\sigma}_2 \colon U \to \mathbb{R}^3$ with the same first fundamental form must have the same area.

true ______ false ______

(iv) An orientable surface patch $\boldsymbol{\sigma}(u, v)$ satisfies $\boldsymbol{\sigma}_u \times \boldsymbol{\sigma}_v \neq 0$ at every point.

true ______ false ______

(b) [**4 pts**] A projective transformation $L : \mathbb{P}^3 \to \mathbb{P}^3$ is represented by the matrix

$$\begin{pmatrix} a & b & c & d \\ e & f & g & h \\ i & j & k & l \\ m & n & o & p \end{pmatrix}.$$

Under what condition(s) is L an affine transformation?

(c) [**4 pts**] In homogeneous coordinates describe the intersection of these two lines:

$$3x - y = 1 \qquad \text{and} \qquad -6x + 2y = 4.$$

(d) [**4 pts**] The homogeneous transformation matrix

$$\begin{pmatrix} -\frac{1}{2} & \frac{\sqrt{3}}{2} & 0 \\ \frac{\sqrt{3}}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

represents a

______ translation
______ rotation
______ reflection

(e) [**6 pts**] An object has its body frame initially aligned with the world frame. Then it rotates about the body y-, x-, and z-axes sequentially by the angles α, β, γ, respectively. Describe the overall transformation as a product of matrices determined by the three Euler angles. (Note that a point is represented as a column vector.)

(f) [**4 pts**] The plane curve $\boldsymbol{\alpha}(t)$ has a simple vertex at $t = a$ if

(g) [(**6 pts**)] Let $\boldsymbol{\alpha}(t)$ be a curve in $\mathbb{R}^3$. Denote by v, κ, τ, T, N, B its speed, curvature, torsion, tangent, principal normal, and binormal, respectively. Give the Frenet formulas that describe the curve.

(h) [**4 pts**] The total Gaussian curvature of an ellipsoid $\frac{x^2}{4} + \frac{y^2}{9} + \frac{z^2}{16} = 1$ is ______.

2. [**14 pts**] *Perspective Projection*

A projection with viewpoint $\boldsymbol{v}$ and viewplane vector $\boldsymbol{n}$, all in homogeneous coordinates, is characterized by the matrix $M = \boldsymbol{v}\boldsymbol{n}^T - (\boldsymbol{n} \cdot \boldsymbol{v})I_4$, where I_4 is the 4×4 identity matrix. Consider the perspective projection from the viewpoint $(1, 1, 2)$ onto the viewplane $z - y = 3$, all in Cartesian coordinates.

(a) [**6 pts**] Compute the matrix M describing the above projection.

(b) [**8 pts**] Find the vanishing points in the three principal directions.

3. [**16 pts**] *Quaternions and Rotations*

Two rotations are carried out one after another. The first rotation is about the vector $(1, 1, 0)$ through an angle of $\frac{\pi}{2}$. It transforms a point $\boldsymbol{v}$ into $\boldsymbol{v}'$. The second rotation is about the vector $(0, -1, 0)$ through an angle of $\frac{3\pi}{2}$. It transforms the point $\boldsymbol{v}'$ into $\boldsymbol{v}''$.

(a) [**6 pts**] Give the quaternions p and q that describe the first and second rotations, respectively.

(b) [**10 pts**] Determine the quaternion that represents the composite rotation from $\boldsymbol{v}$ to $\boldsymbol{v}''$. What are the axis $\boldsymbol{u}$ and the angle θ of the composite rotation? [Use atan2(s, c) to denote the angle $\alpha \in [0, 2\pi)$ such that $\sin\alpha = s$ and $\cos\alpha = c$.]

4. [**12 pts**] *Space Curves*

Compute the curvature κ, torsion τ, unit tangent T, principal normal N, and binormal B for the following curve:

$$\boldsymbol{\alpha}(t) = \left(\frac{4}{5} \cos t, 1 - \sin t, -\frac{3}{5} \cos t \right).$$

5. [**14 pts**] *Surfaces*

Calculate the Gaussian curvature of the surface

$$\boldsymbol{\sigma}(u, v) = (u + v, u - v, uv)$$

at the point $(2, 0, 1)$.