

Iowa State University  
Department of Computer Science  
Machine Learning (Com S 573)  
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Spring 2007  
Due March 9, 2007  
Problem Set 4

1. **(25 pts.)** Prove the modularity property of kernel functions (see lecture notes for statement of this property).
2. **(25 pts.)** Consider the problem of solving for the lagrange multipliers  $\alpha_1$  and  $\alpha_2$  that define the weight vector corresponding the maximum margin separating hyperplane and the bias term  $b$  in the special case where there are only 2 linearly separable training samples  $(\mathbf{X}_1, y_1)$  and  $(\mathbf{X}_2, y_2)$ . Precisely formulate the corresponding optimization problem. Does this optimization problem have a closed-form solution? Comment on the possibility of using the results of your analysis for solving the general SVM optimization problem where the training set contains more than 2 training examples.
3. **25 pts.)** Show that the VC-dimension of convex  $d$ -gons (polygons with  $d$  sides in 2-dimensional plane is  $2d + 1$ .
4. **25 pts.)** For any two hypothesis classes  $H_1$  and  $H_2$  with VC-dimensions  $d_1$  and  $d_2$  respectively, show that the VC-dimension  $H_1 \cup H_2$  is less than or equal to  $d_1 + d_2 + 1$ .
5. **25 pts.)** How would you generalize the  $p$ -spectrum kernel for sequences to work with 2-dimensional images? Design a suitable kernel for classifying documents that consist of both text and images. (Hint: use modularity property of kernels).