**Principles of Artificial Intelligence**

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Problem set 4
Due October 5, 2007

Note: The problems marked with ** are targeted primarily to students enrolled in ComS 572; Others are of course encouraged to solve such problems for extra credit.

1. (20 pts). Solve Problem 7.4 from the Russell and Norvig text.
2. (20 pts.) Solve Problem 7.7 from the Russell and Norvig text.
3. (20 pts.) Solve Problem 7.8 from the Russell and Norvig text.
4. (20 pts.) Solve Problem 8.11 from the Russell and Norvig text.
5. (20 pts.) Solve Problem 8.13 from the Russell and Norvig text.
6. (20 pts.)
   (a) *Prove* that for any sentence p and any object a, $\forall x\ p(x) \models p(a)$. Identify the conditions (if any) under which $p(a) \models \forall x\ p(x)$.
   (b) *Prove* that $\neg\exists x\ p(x)$ and $\forall x\ \neg p(x)$ are equivalent (this gives us the de Morgan's law for FOPL).
7. (20 pts.) Consider a First-order logical system which uses two 1-place predicates namely, Big and Small. The set of object constants is given by a, b. Enumerate all possible Models in this case. For each of the following sentences, identify the models in which the given sentence is true.
   (a) $\text{Big}(a) \wedge \text{Big}(b)$
   (b) $\text{Big}(a) \vee \text{Big}(b)$
   (c) $\forall x\ \text{Big}(x)$
   (d) $\forall x\ \neg\text{Big}(x)$
   (e) $\exists x\ \text{Big}(x)$
   (f) $\exists x\ \neg\text{Big}(x)$

(g) $\forall$x Big(x) $\wedge$ Small(x)

(h) $\forall$x Big(x) $\vee$ Small(x)

(i) $\forall$x Big(x) $\Rightarrow$ $\neg$Small(x)

8. (20 pts.) Determine whether the expressions $p$ and $q$ unify with each other in each of the following cases. If so, give the most general unifier; If not, give a brief explanation (Assume that the upper case letters are (object, predicate, or function) constants and that the lower case letters are variables).

   (a) $p =$ F(G(v), H(u,v)); $q =$ F(w, J(x,y))

   (b) $p =$ F(x, F(u,x)); $q =$ F(F(y,A), F(z, F(B,z)))

   (c) $p = \mathtt{F}(\mathtt{x}_1, \mathtt{G}(\mathtt{x}_2, \mathtt{x}_3), \mathtt{x}_2, \mathtt{B})$; $q = \mathtt{F}(\mathtt{G}(\mathtt{H}(\mathtt{A}, \mathtt{x}_5), \mathtt{x}_2), \mathtt{x}_1, \mathtt{H}(\mathtt{A}, \mathtt{x}_4), \mathtt{x}_4)$

9. (20 pts.) Given two binding lists $\sigma_1$ and $\sigma_2$, we will say that $\sigma_1$ and $\sigma_2$ are *compatible* if there is a binding list $\sigma$ that is *less general than or equal to* both $\sigma_1$ and $\sigma_2$. We will denote the most general such $\sigma$ by $\sigma_1 \star \sigma_2$.

   (a) In each of the following cases, determine if $\sigma_1$ and $\sigma_2$ are compatible and compute $\sigma_1 \star \sigma_2$ if they are:

      i. $\sigma_1 =$ (Mary / x); $\sigma_2 =$ (John / y)

      ii. $\sigma_1 =$ (Mary / x); $\sigma_2 =$ (John / x)

      iii. $\sigma_1 =$ (y/x); $\sigma_2 =$ (house-of(x) / y)

      iv. $\sigma_1 =$ (); $\sigma_2$ is some arbitrary binding list

   (b) ** (20 pts). Show that $\sigma_1$ and $\sigma_2$ are compatible *if and only if* for every expression $p$, $p|\sigma_1$ and $p|\sigma_2$ can be unified.

10. (20 pts.) Put the following FOPL formulae into clause form (CNF):

   (a) $\forall$x $\forall$y ((P(x) $\wedge$ Q(y)) $\Rightarrow$ $\exists$z R(x, y, z))

   (b) $\exists$x $\forall$y $\exists$z (P(x) $\Rightarrow$ (Q(y) $\Rightarrow$ R(z)))

11. (20 pts.) Consider the following information:

   - Animals can outrun any animals that they eat.
   - Carnivores eat other animals.
   - Outrunning is transitive i.e., if x can outrun y and y can outrun z, then x can outrun z.
   - Lions eat zebras.

- Zebras can outrun dogs.
- Dogs are carnivores.

(a) Translate each of the above sentences into FOPL.

(b) Translate the resulting set of FOPL sentences into clause form (CNF).

(c) Use resolution theorem proving using the set of support strategy and Green's trick for answer extraction to find two animals that lions can outrun.

12. ** (40 pts.) Solve problem 9.1 from the Russell and Norvig text .