Homework 1
Computer Science 511
Fall 2009
Due in class on Friday, September 4

Reading Assignment

Kleinberg and Tardos, Chapter 7.

Problem Set

In this and all subsequent assignments, unless otherwise stated, all problems are from the Kleinberg and Tardos text (K & T).

1. (10 points) Suppose that a network $G = (V, E)$ has some infinite-capacity arcs, but no infinite-capacity paths from s to t. Let $E_0 \subseteq E$ denote the set of arcs with finite capacities. Show that we can replace the capacity of each infinite-capacity arc by a finite number $M \geq \sum_{e \in E_0} c(e)$ without affecting the maximum flow value.

2. (10 points) Show that the number of minimum cuts in a flow network is not polynomially-bounded by constructing a family of networks where the number of minimum cuts grows exponentially in n, the number of vertices.

3. (10 points) Describe an efficient algorithm to determine if a flow network has a unique minimum cut.

   Hint: Design two algorithms:

   (i) An algorithm to find a minimum cut (A, B) such that for every other minimum cut (A', B'), $A \subseteq A'$.

   (ii) An algorithm to find a minimum cut (A, B) such that for every other minimum cut (A', B'), $A' \subseteq A$.

4. (10 points) Let $G = (V, E)$ be a flow network with integer arc capacities. A flow f in G is *even* if for every arc $e \in E$, $f(e)$ is an even number; it is *odd* if for every $e \in E$, $f(e)$ is an odd number. For each of the following statements, either prove that the claim is true or give a counterexample for it:

   (a) If all arc capacities are even, then G has an even maximum flow.

   (b) If all arc capacities are odd, then G has an odd maximum flow.

5. (10 points) Let $(A, V - A)$ and $(A', V - A'')$ be two s-t cuts in a flow network $G = (V, E)$. Show that the cut capacity function is *submodular*; that is,

$$c(A, V - A) + c(A', V - A') \geq c(A \cup A', V - (A \cup A')) + c(A \cap A', V - (A \cap A')).$$

6. (10 points) Exercise 13, page 420–421 of K & T.

Note. We reserve the right to grade only a subset of the problems assigned. Which problems will be graded will be decided after the submission deadline.