

Com S 331  
Fall 2009  
Homework #10

Due Wednesday, December 2 at beginning of class.

In problems 73-79, write a BRL program that computes the indicated function. You may make free use of macros that we have developed in class and/or macros that you develop here.

73. The reverse function  $\mathbf{rev} : \{0, 1\}^* \rightarrow \{0, 1\}^*$  defined by the following recursion.

$$\mathbf{rev}(\lambda) = \lambda.$$

$$\text{For } x \in \{0, 1\}^* \text{ and } b \in \{0, 1\}, \mathbf{rev}(bx) = \mathbf{rev}(x)b.$$

74. The concatenation function  $\mathbf{conc} : (\{0, 1\}^*)^2 \rightarrow \{0, 1\}^*$  defined by  $\mathbf{conc}(u, v) = uv$ .

75. The function  $\mathbf{addbits} : \{0, 1\}^* \rightarrow \{0, 1\}^*$  defined by the following recursion.

$$\text{For all } x \in \{\lambda, 0, 1\}, \mathbf{addbits}(x) = x.$$

$$\text{For all } x \in \{0, 1\}^* \text{ and } a, b \in \{0, 1\}, \mathbf{addbits}(abx) = a \mathbf{addbits}(x).$$

76. The bit-doubling function  $\mathbf{bd} : \{0, 1\}^* \rightarrow \{0, 1\}^*$  defined by the following recursion.

$$\mathbf{bd}(\lambda) = \lambda.$$

$$\text{For all } x \in \{0, 1\}^* \text{ and } b \in \{0, 1\}, \mathbf{bd}(xb) = \mathbf{bd}(x)bb.$$

77. The string-pairing function  $\langle, \rangle : (\{0, 1\}^*)^2 \rightarrow \{0, 1\}^*$  defined by  $\langle u, v \rangle = \mathbf{bd}(u)01v$ .

78. The projection partial function  $\mathbf{proj}_1 : \{0, 1\}^* \dashrightarrow \{0, 1\}^*$  defined by

$$\mathbf{proj}_1(u) = \begin{cases} x & \text{if } u = \langle x, y \rangle \\ \uparrow & \text{if } u \text{ is not of this form.} \end{cases}$$

79. The projection partial function  $\mathbf{proj}_2 : \{0, 1\}^* \dashrightarrow \{0, 1\}^*$  defined by

$$\mathbf{proj}_2(u) = \begin{cases} y & \text{if } u = \langle x, y \rangle \\ \uparrow & \text{if } u \text{ is not of this form.} \end{cases}$$

80. Define the *characteristic function* of a language  $A \subseteq \{0, 1\}^*$  to be the function  $\chi_A : \{0, 1\}^* \rightarrow \{0, 1\}$  defined by

$$\chi_A = \begin{cases} 1 & \text{if } x \in A \\ 0 & \text{if } x \notin A. \end{cases}$$

Prove: If  $A \subseteq \{0, 1\}^*$  is regular, then  $\chi_A$  is BRL computable.