Com S 331 Name KEY

Fall, 2009

Exam 1

This is a closed-book, closed-notes, no-calculator, no-cellphone, individual-effort examination. All answers should be justified, at least briefly. Please do all your work on these pages.

Problems 1, 2, 3, and 4 all use the language

$$A = \{ x \in \{0,1\}^* \mid \#(1,x) \geq 4 \},$$

where $\#(1,x)$ is the number of 1's in x. For each $x \in A$, we also use the quantities

$f(x)$ = the number of 0's between the <u>first</u> <u>two</u> 1's in x

and

$g(x)$ = the number of 0's between the <u>last</u> <u>two</u> 1's in x.

For example, if $x = 0100011011$, then $x \in A$, $f(x) = 3$, and $g(x) = 0$.

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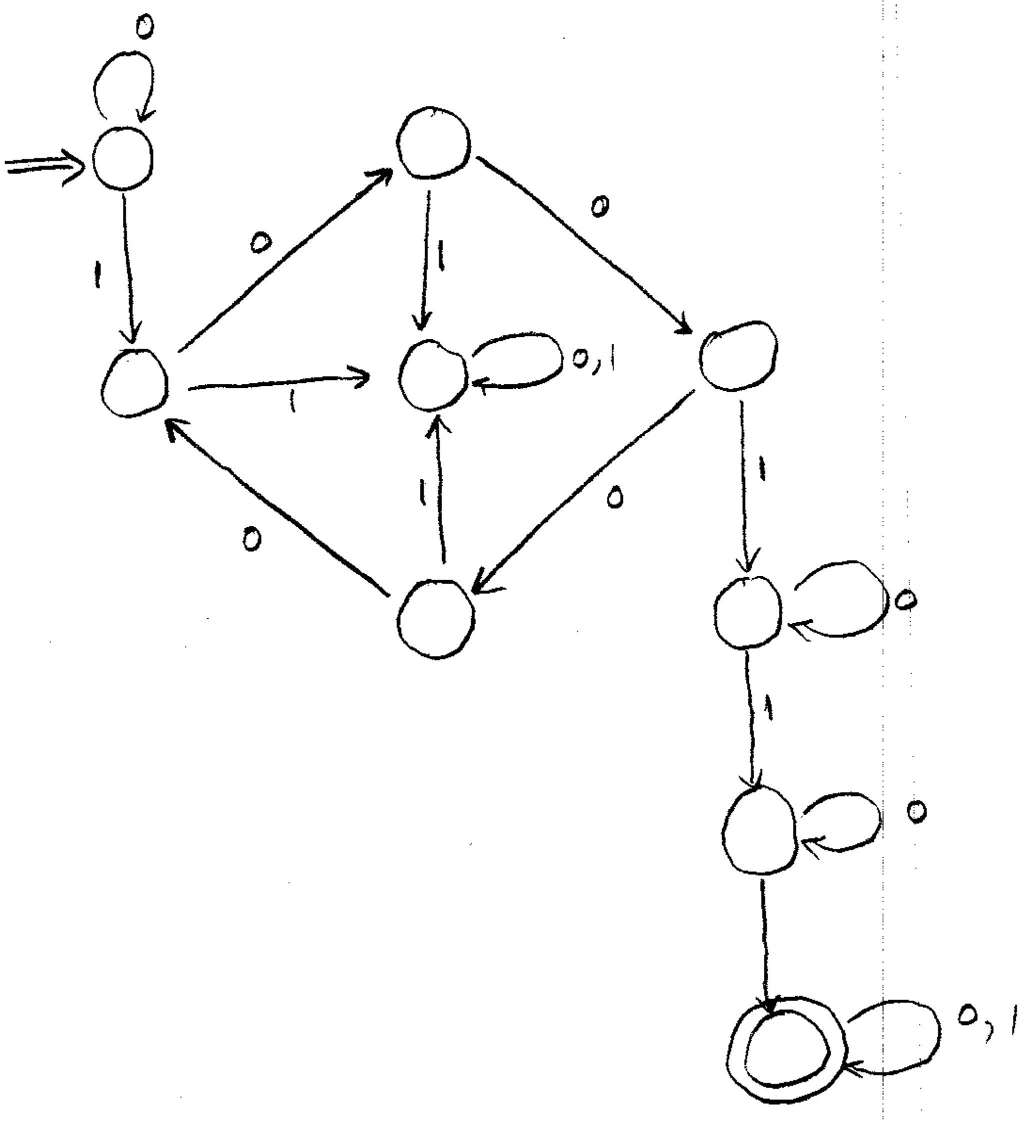
1. (15 points) Design a DFA that decides the language
$B = \{x \in A \mid f(x) \text{ is divisible by } 2, \text{ but not by } 4\}$.



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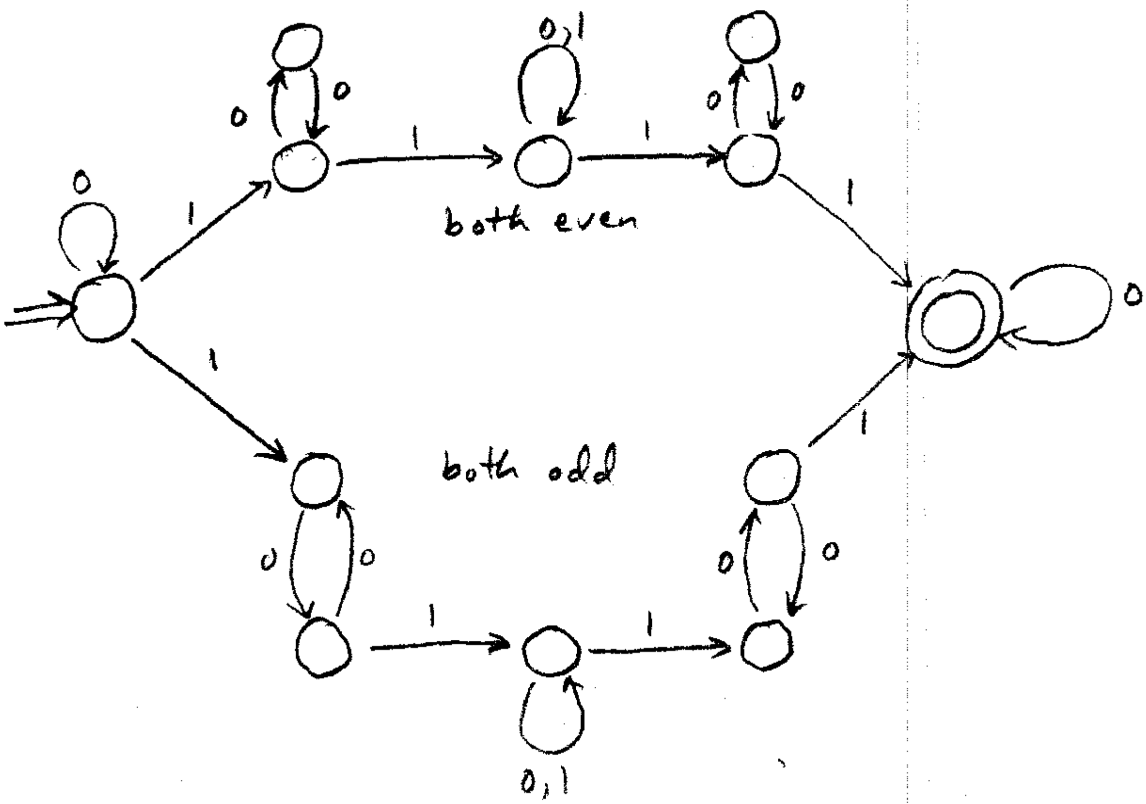
2. (15 points) Design an NFA that decides the language

$$C = \{x \in A \mid f(x) + g(x) \text{ is even}\}.$$

$$= \{x \in A \mid f(x) \text{ and } g(x) \text{ are both even or both odd}\}.$$



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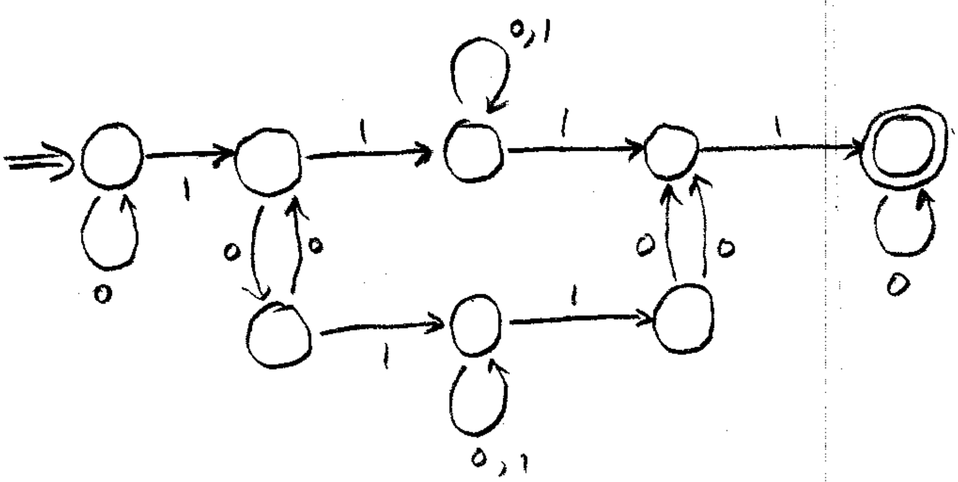
Alternative NFA with fewer states :



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3. (15 points) Give a regular expression α that denotes the language C of problem 2.

Let

$$\beta = (00)^* 1 (0+1)^* 1 (00)^*.$$

Then

$$\alpha = 0^* 1 \left(\beta + 0\beta 0\right) 1 0^*.$$

(Clear from inspection of NFAs in problem 2.)

Optional simplification:

Let

$$\gamma = 1(0+1)^* 1.$$

Then

$$\beta = (00)^* \gamma (00)^*$$

and

$$0\beta 0 = (00)^* 0 \gamma 0 (00)^*,$$

so

$$\alpha = 0^* 1 (00)^* \left(\gamma + 0\gamma 0\right) (00)^* 1 0^*.$$

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4. (20 points) Prove that the language
$$D = \{x \in A \mid f(x) = g(x)\}$$
is not regular.

Proof. Let R be a regular language such that $D \subseteq R$. It suffices to prove that $R \neq D$.

Choose a positive integer k for R as in the pumping lemma. Let
$$x = 10^k 11,\ y = 0^k,\ z = 1.$$
Then $xyz = 10^k 110^k 1 \in D \subseteq R$ and $|y| \geq k$, so the pumping lemma tells us that we can write $y = 0^p 0^q 0^r$, where $q > 0$, $p+q+r = k$, and (taking $i=0$), $x0^p0^rz \in R$. But
$$x0^p0^rz = 10^k 110^{p+r}1 \notin D$$
because $k-(p+r) = q > 0$. We now have $x0^p0^rz \in R - D$, so $R \neq D$. $\square$

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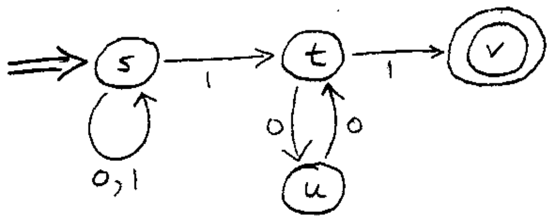
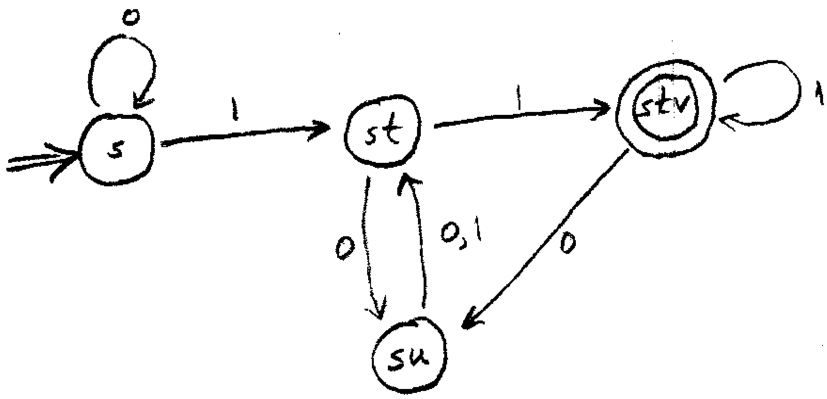
5. (20 points) Design a DFA that is equivalent to the following NFA.





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6. (15 points) Given a language $S \subseteq \{0,1\}^*$ and a string $x \in \{0,1,2\}^*$, define an S-instance of x to be a string $y \in \{0,1\}^*$ obtained from x by substituting an element of S for each 2 in x.

(Example: If $S = \{\lambda, 100\}$, then the string $x = 0212$ has 4 S-instances, namely,

$0\underline{\lambda}1\underline{\lambda} = 01$,

$0\underline{\lambda}1\underline{100} = 01100$,

$0\underline{100}1\underline{\lambda} = 01001$,

$0\underline{100}1\underline{100} = 0100 1100$.)

Let $R \subseteq \{0,1,2\}^*$ and $S \subseteq \{0,1\}^*$, and let

$T = \{ y \in \{0,1\}^* \mid y$ is an S-instance of some string in $R\}$.

Prove: If R and S are regular, then T is regular.

Proof. Assume the hypothesis. Then there are regular expressions α_R and α_S denoting R and S, respectively. Let α_T be the regular expression obtained from α_R by substuting α_S for each 2. Then α_T denotes T, so T is regular. $\square$

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