Com S 331 Name KEY

Spring, 2005

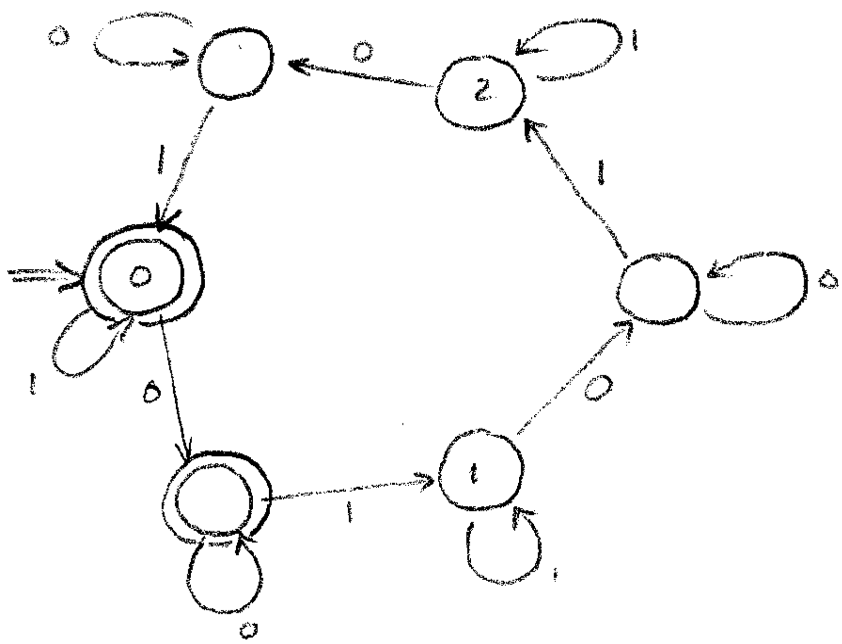
Exam 1

This is a closed-book, closed-notes, no-calculator, no-cellphone, individual-effort examination. All answers should be justified, at least briefly. Please do all your work on these pages.

1. (15 points) Let

$$A = \left\{ x \in \{0,1\}^* \;\middle|\; \begin{array}{l}\text{the number of occurrences of 01 as}\\ \text{a substring of } x \text{ is a multiple of 3}\end{array} \right\}.$$

Design a DFA M such that $L(M) = A$.



Name KEY

2. (15 points) Give a regular expression that denotes the language A of problem 1.

$$1^*0^*(0 \mid 1^*0^*0 \mid 1 \mid 1^*0^*0 \mid 1 \mid 1^*0^*)^*$$

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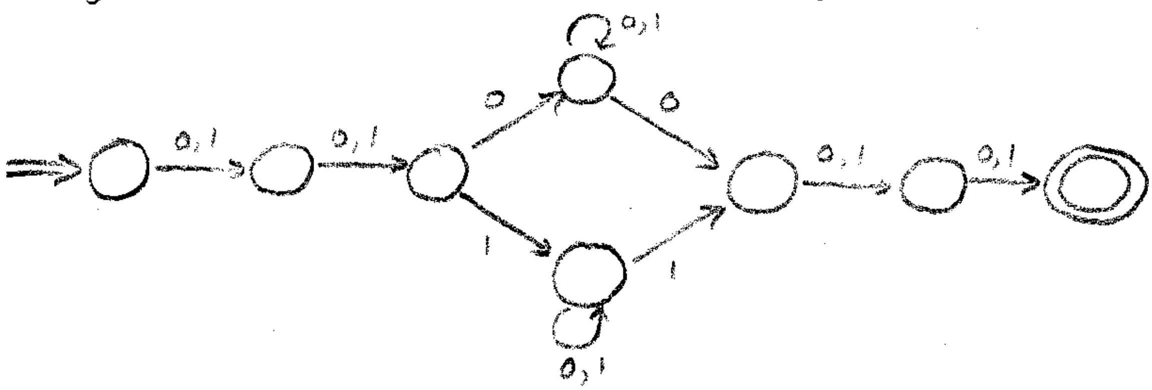
3. (15 points) Let

$$B = \left\{ x \in \{0,1\}^* \;\middle|\; \begin{array}{l} |x| \geq 6, \text{ and the third bit and the} \\ \text{third-to-last bit of } x \text{ are the same} \end{array} \right\}.$$

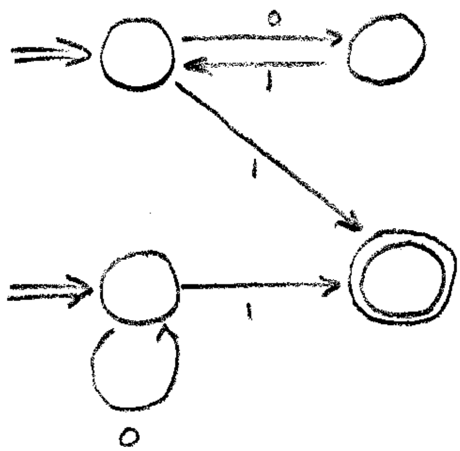
(For example, 110011000 ∈ B because these bits are the same.)

Design an NFA N such that L(N) = B.



Name KEY

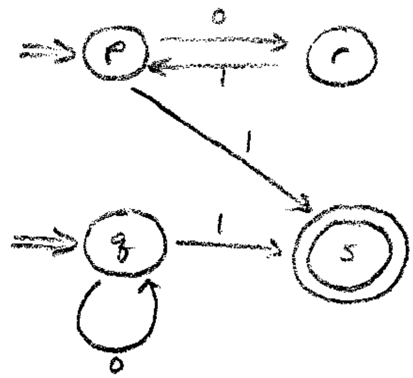
4. (15 points) Let $\alpha = ((01)^* + 0^*)1$. Design an NFA N such that $L(N) = L(\alpha)$.



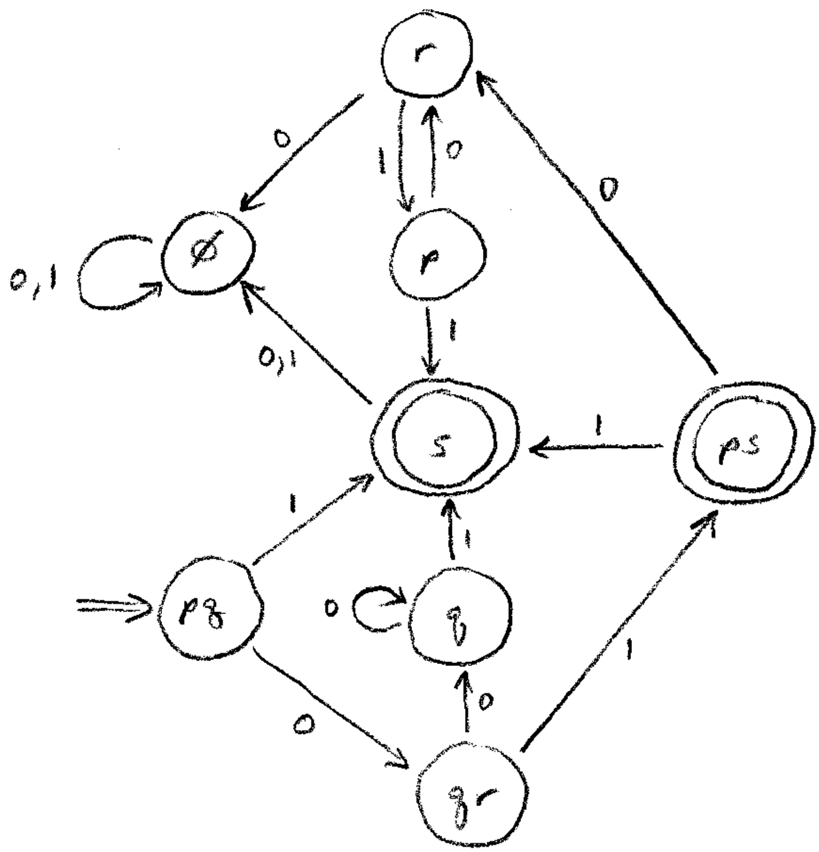
Name KEY

5. (15 points) As in problem 4, let $\alpha = ((01)^* + 0^*)1$.
Design a DFA M such that $L(M) = L(\alpha)$.

Applying subset construction to



gives the DFA



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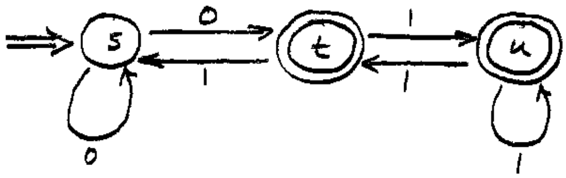
6. (15 points) Let N be the NFA



Give a regular expression β such that $L(\beta) = L(N)$.

We want the expression $\beta = \alpha_{st}^{stu} + \alpha_{su}^{stu}$.

We have

$$\alpha_{st}^{stu} = \alpha_{st}^{su} + \alpha_{st}^{su}\left(\alpha_{tt}^{su}\right)^* \alpha_{tt}^{su} = \gamma + \gamma\delta^*\delta$$

$$= \gamma(\lambda + \delta^*\delta) = \gamma\delta^*$$

and

$$\alpha_{su}^{stu} = \alpha_{su}^{su} + \alpha_{st}^{su}\left(\alpha_{tt}^{su}\right)^* \alpha_{tu}^{su} = \theta + \gamma\delta^*\tau,$$

so

$$\beta = \gamma\delta^* + \theta + \gamma\delta^*\tau = \theta + \gamma\delta^*(\lambda + \tau),$$

where

$$\gamma = \alpha_{st}^{su} = 00^*, \quad \delta = \alpha_{tt}^{su} = 10^*0 + 11^*1 + \lambda$$

$$= 1(00^* + 11^*) + \lambda$$

$$\theta = \alpha_{su}^{su} = \emptyset, \quad \tau = \alpha_{tu}^{su} = 11^*.$$

We thus have

$$\beta = \emptyset + 00^*\left(1(00^* + 11^*) + \lambda\right)^*(\lambda + 11^*)$$

$$= 00^*\left(1(00^* + 11^*)\right)^* 1^*$$

Name KEY

7. (10 points) For each $n \in \mathbb{N}$, define the string $x_n \in \{0,1\}^*$ by the recursion

$$x_0 = \lambda, \quad x_{n+1} = x_n 0^n 1.$$

(a) What are the strings x_1, x_2, x_3, and x_4?

$x_1 = 1$ $\quad$ $x_3 = 101001$

$x_2 = 101$ $\quad$ $x_4 = 1010010001$

(b) Prove that the language $C = \{x_n \mid n \in \mathbb{N}\}$ is not regular.

Let D be a regular language such that $C \subseteq D$. It suffices to show that $D \neq C$. Choose $k \in \mathbb{N}$ for D as in the pumping lemma. Let $x = x_k$, $y = 0^k$, $z = 1$. Then $xyz = x_k 0^k 1 = x_{k+1} \in C \subseteq D$ and $|y| \geq k$, so the pumping lemma tells us that y can be written in the form $y = 0^k = 0^a 0^b 0^c$, where $b > 0$ and (taking $i = 0$) $x 0^a 0^c z = x_k 0^{a+c} 1 \in D$. Since $a + c < k$ (because $b > 0$), $x_k 0^{a+c} 1 \notin C$. We thus have $x_k 0^{a+c} 1 \in D - C$, so $D \neq C$. $\square$

Name KEY

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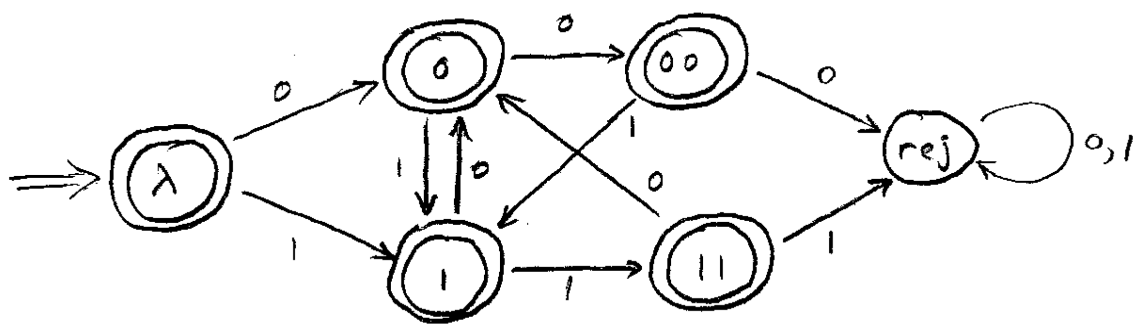
Spring, 2004
Exam 1

Name KEY

This is a closed-book, closed-notes, no-calculator, individual-effort examination. All answers should be justified, at least briefly. Please do all your work on these pages.

1. (15 points) Design a DFA M such that
$L(M) = \{x \in \{0,1\}^* \mid \text{no three consecutive bits of } x \text{ are the same}\}$.



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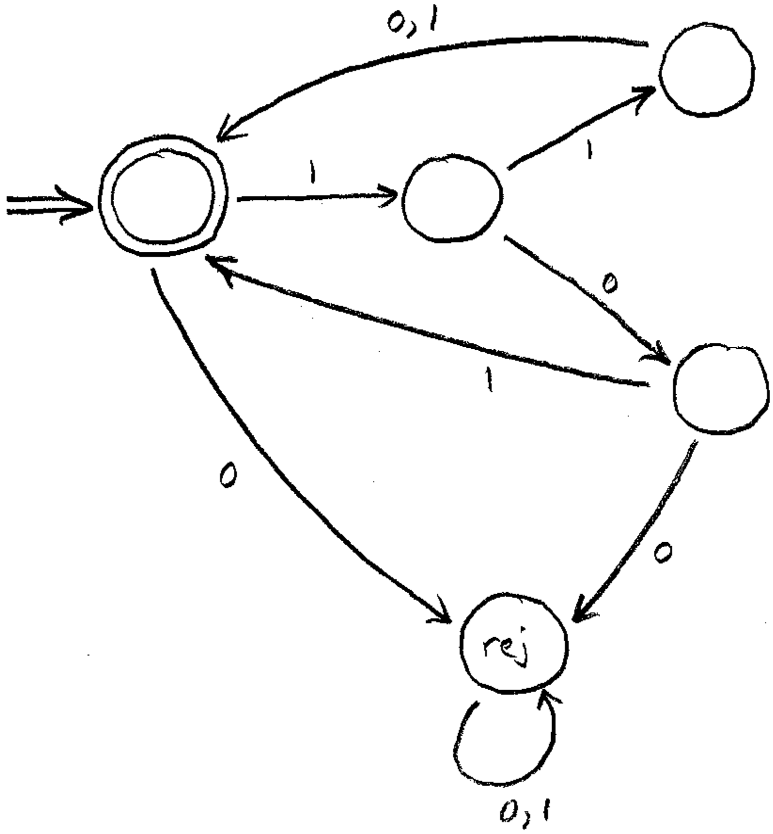
2. (10 points) Design a DFA that is equivalent to the regular expression $(1(01+10+11))^*$.



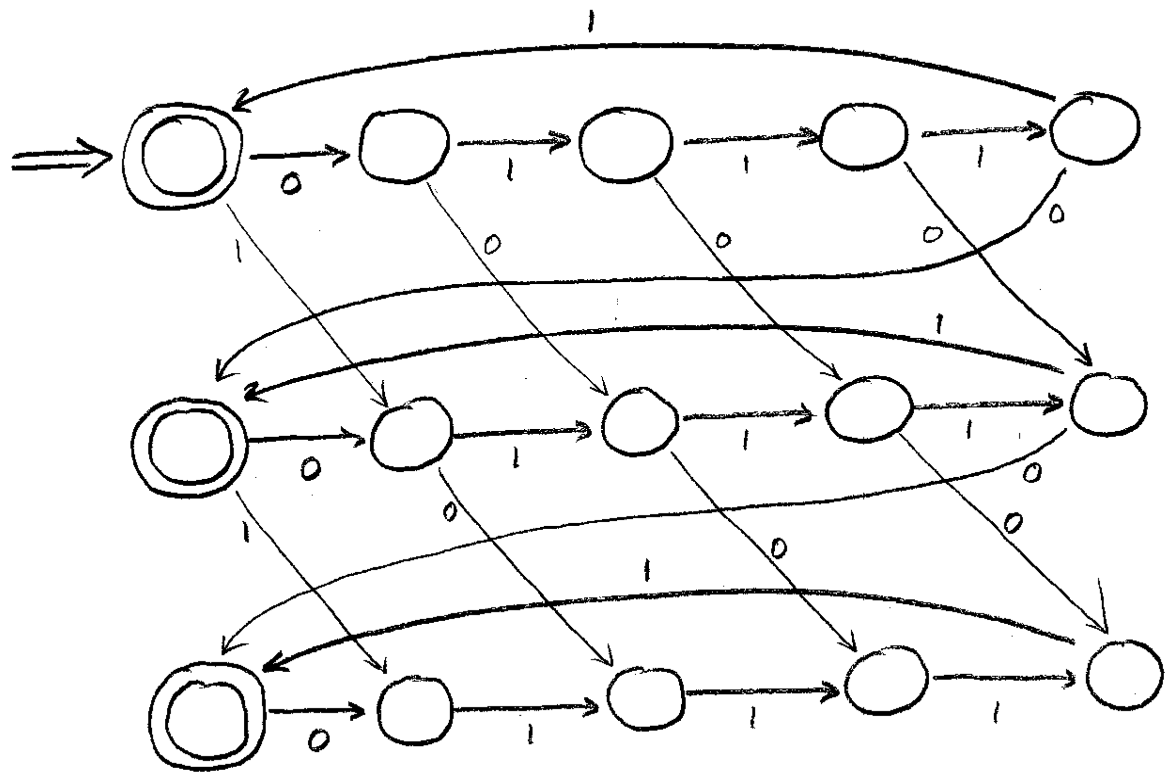
Name KEY

3. (15 points) Recall: If $A \subseteq \{0,1\}^*$ and $k \in \mathbb{N}$, then $N_k(A)$ is the set of all $x \in \{0,1\}^*$ such that there exists $y \in A$ such that $|y| = |x|$ and $d_H(x,y) \le k$, where

$d_H(x,y)$ = the Hamming distance from x to y
= the number of positions at which x and y disagree.

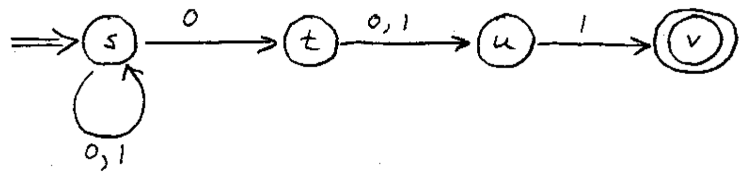
(The text uses the notation $H(x,y)$ for $d_H(x,y)$.)

Let $A = L((01111)^*)$. Give an NFA N such that $L(N) = N_2(A)$.



Name KEY

4. (15 points) Find a DFA equivalent to the NFA



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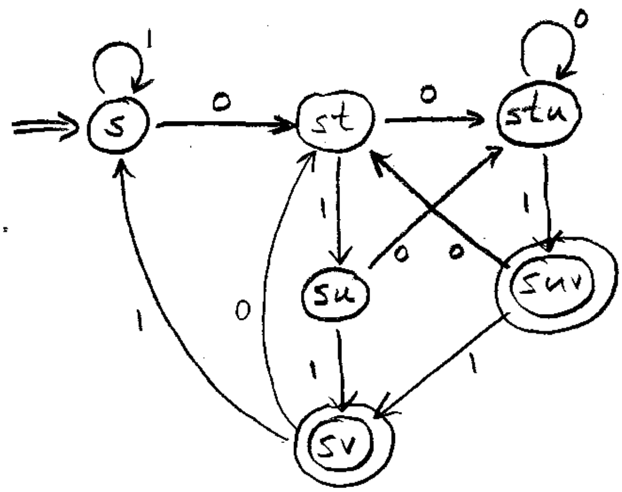


Name KEY

5. (15 points) Let A be the set of all strings $x \in \{0,1\}^*$ such that $\#(1, x)$ is divisible by (at least one of) 2, 3, or 5. Find a regular expression α such that $L(\alpha) = A$.

$$\alpha = (\beta_2^* + \beta_3^* + \beta_5^*)\, 0^*,$$

where

$$\beta_2 = 0^* 1\, 0^* 1,$$

$$\beta_3 = 0^* 1\, 0^* 1\, 0^* 1,$$

$$\beta_5 = 0^* 1\, 0^* 1\, 0^* 1\, 0^* 1\, 0^* 1.$$

Name KEY

6. (15 points) Prove that the language

$$A = \{0^s 1^t \mid s-3 \le t \le s+3\}$$

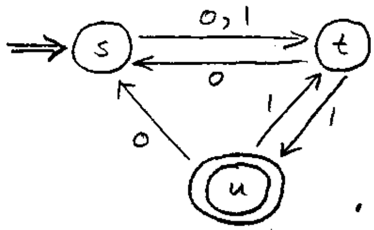
is not regular.

Let B be a regular language such that $A \subseteq B \subseteq \{0,1\}^*$. It suffices to show that $B \ne A$.

Choose $k \in \mathbb{Z}^+$ for B as in the Pumping Lemma. Let $x = 0^k$, $y = 1^k$, $z = \lambda$. Then $xyz = 0^k 1^k \in A$ and $|y| \ge k$, so the Pumping Lemma tells us that there exist $p, q, r \in \mathbb{N}$ such that $p+q+r = k$, $q > 0$, and (taking $i = 5$) $x 1^{p+5q+r} z = 0^k 1^{k+4q} \in B$. However, $k + 4q \ge k+4 > k+3$, so $0^k 1^{k+4q} \notin A$. We thus have $0^k 1^{k+4q} \in B - A$, so $B \ne A$. □

Name KEY

7. (15 points) Find a regular expression α such that $L(\alpha) = L(M)$, where M is the DFA



$$\alpha = \alpha_{su}^{stu} = \alpha_{su}^{su} + \alpha_{st}^{su} (\alpha_{tt}^{su})^* \alpha_{tu}^{su},$$

where

$$\alpha_{su}^{su} = \emptyset,$$

$$\alpha_{st}^{su} = 0+1,$$

$$\alpha_{tt}^{su} = \lambda + 0(0+1) + 11 + 10(0+1)$$

$$\alpha_{tu}^{su} = 1.$$

Thus

$$\alpha = \emptyset + (0+1)(\lambda + 00 + 01 + 11 + 100 + 101)^* 1$$

$$= (0+1)(00+01+11+100+101)^* 1.$$

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Exam Sample Questions

Com S 331

March 20, 2007

1. (15 points) Let

$$A = \{x \in \{0,1\}^* \mid \text{the number of occurences of 01 as a substring of } x \text{ is a multiple of } 3\}$$

Design a DFA M such that $L(M) = A$.

Solution.

	0	1
$\Rightarrow F0$	a	0
Fa	a	1
1	b	1
b	b	2
2	c	2
c	c	0

2. (15 points) Find a regular expression α such that $L(\alpha) = L(M)$, where M is the DFA given in the following transition table.

	0	1
⇒s	t	t
t	s	u
*F*u	s	t

3. (15 points) Prove that the language

$$A = \{0^s 1^t \,|s - 3 \leq t \leq s + 3\}$$

is not regular.

Solution.

Let B be a regular set such that $A \subseteq B$. It suffices to show that $B \neq A$. Choose $k \in \mathbb{N}$ for B as in the pumping lemma. Let $x = 0^k$, $y = 1^k$, $z = \lambda$. Then $xyz \in A$ and $|y| \geq k$, so the pumping lemma tells us that there exist $p, q, r \in \mathbb{N}$ such that $p + q + r = k$, $q > 0$ and (taking $i = 5$) $x1^{p+5q+r}z = 0^k 1^{k+4q} \in B$. However, $k + 4q \geq k + 4 > k + 3$, so $0^k 1^{k+4q} \notin A$. We thus have $B \neq A$.

4. Let A be the set of all strings $x \in \{0,1\}^*$ such that $\#(1,x)$ is divisible by (at least one of) 2,3, or 5. Find a regular expression α such that $L(\alpha) = A$.

Solution.

$\alpha = (\beta_2^* + \beta_3^* + \beta_5^*)\, 0^*$, where

$$\beta_2 = 0^*10^*1,$$

$$\beta_3 = \beta_2 \cdot 0^*1,$$

$$\beta_5 = \beta_3 \cdot 0^*10^*1.$$

5. (15 points) Find a DFA equivalent to the NFA given in the following transition table.

	0	1
⇒s	s,t	s
t	u	u
u	∅	v
*F*v	∅	∅

Solution.

	0	1
⇒ s	st	s
st	stu	su
su	stu	sv
*F*sv	∅	s
stu	stu	suv
*F*suv	st	sv

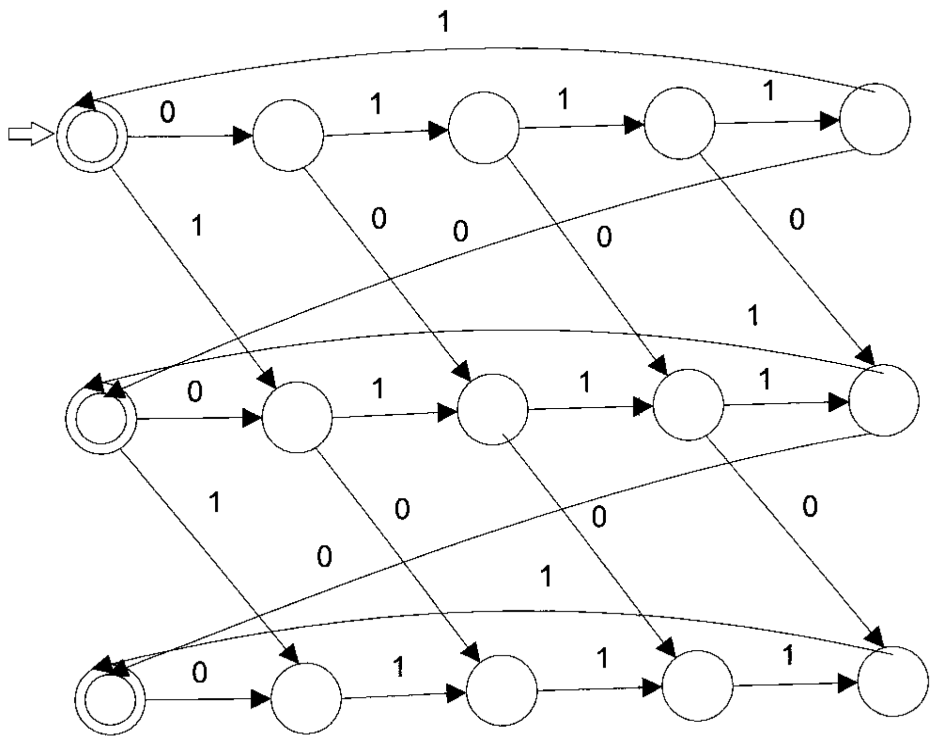
6. (15 points) Recall: If $A \subseteq \{0,1\}^*$ and $k \in \mathbb{N}$, then $N_k(A)$ is the set of all $x \in \{0,1\}^*$ such that there exists $y \in A$ such that $|y| = |x|$ and $d_H(x,y) \leq k$, where

$$\begin{aligned} d_H(x,y) &= \text{the Hamming distance from } x \text{ to } y \\ &= \text{the number of positions at which } x \text{ and } y \text{ disagree.} \end{aligned}$$

(The text uses the notation $H(x,y)$ for $d_H(x,y)$.)

Let $A = L((01111)^*)$. Give an NFA N such that $L(N) = N_2(A)$.

Solution.



7. (10 points) Design a DFA that is equivalent to the regular expression

$$(1(01+10+11))^* .$$

Solution.

	0	1
$\Rightarrow F\lambda$	rej	1
1	10	11
10	rej	λ
11	λ	λ
rej	rej	rej

8. (15 points) Design a DFA M such that

$$L(M) = \{x \in \{0,1\}^* | \text{no three consecutive bits of } x \text{ are the same}\}.$$

Solution.

	0	1
$\Rightarrow F\ \lambda$	0	1
$F0$	00	1
$F1$	0	11
$F00$	rej	1
$F11$	0	rej
rej	rej	rej

9. (15 points) Give a regular expression that denotes the language A of problem 1.

Solution.

$1^*0^*(011^*0^*011^*0^*011^*0^*)^*$

10. (15 points) Let

$$B = \{x \in \{0,1\}^* \mid |x| \geq 6 \text{ and the third bit and the third-to-last bit of } x \text{ agree}\}.$$

Design an NFA N such that $L(N) = B$.

Solution.

	0	1
⇒0	1	1
1	2	2
2	3a	3b
3a	3a,4	3a
3b	3b	3b,4
4	5	5
5	6	6
$F6$	$\emptyset$	$\emptyset$

11. (15 points) Let $\alpha = ((01)^* + 0^*)1$. Design an NFA N such that $L(N) = L(\alpha)$.

Solution.

	0	1
⇒0a	1a	1b
⇒0b	0b	1b
1a	$\emptyset$	0a
F1b	$\emptyset$	$\emptyset$

12. (15 points) As in problem 12, let $\alpha = ((01)^* + 0^*)1$. Design a DFA M such that $L(M) = L(\alpha)$.

Solution. Simply apply the subset construction to the NFA given in the solution to problem 11.

13. (15 points) Let N be the NFA given in the following transition table.

	0	1
s	s	$\emptyset$
t	$\emptyset$	s,t
Fu	$\emptyset$	t,u

Give a regular expression β such that $L(\beta) = L(N)$.

Solution. $\beta = 00^* \left(1\left(00^* + 11^*\right)\right)^* 1^*$

14. (10 points) For each $n \in \mathbb{N}$, define the string $x_n \in \{0,1\}^*$ by the recursion

$$x_0 = \lambda, x_{n+1} = x_n 0^n 1.$$

(a) What are the strings x_1, x_2, x_3 and x_4?

Solution. $x_1 = 1$, $x_2 = 101$, $x_3 = 101001$, $x_4 = 1010010001$.

(b) Prove that the language $C = \{x_n | n \in \mathbb{N}\}$ is not regular.

Solution. Let D be a regular language such that $C \subseteq D$. It suffices to show that $D \neq C$. Choose $k \in \mathbb{N}$ for D as in the pumping lemma. Let $x = x_k$, $y = 0^k$, $z = 1$. Then $xyz \in C \subseteq D$ and $|y| \geq k$, so the pumping lemma tells us that y can be written in the form $y = 0^k = 0^a 0^b 0^c$, where $b > 0$ and (taking $i = 0$), $x 0^a 0^c z = x_k 0^{a+c} 1 \in D$. Since $a + c < k$, $x_k 0^{a+c} 1 \notin C$, so $D \neq C$.

Good luck!