

1 Topic: Broadcast

2 Hitting Game Reduction

Lemma 1 :

Definition: Let $H_k = P_1, \dots, P_k$, be the global history of what happens through k rounds in the abstract broadcast protocols.

P_i = id of unique node that transmits alone in slot i .

T_i = set of transmitter in slot i

T_i^1 = set of transmitter at round i if they were in S

T_i^0 = set of transmitter at round i if they were not in S

$T_i^j = u | \pi(u, j, H_{i-1}) = 1$

The n^{th} hitting game played on graphs in C_n between explorer and referee, where referee picks a subset $S \subseteq 1, \dots, n$ and explorer doesn't know S , and chooses M_1, M_2, \dots . The explorer is trying to isolate an elements of S . The set of rules they follow are as below,

1. at move i , if $| M_i \cap S | = 1$, game is over and Explorer wins the game (i.e; $M_i = T_i^1$)
else
2. if $| M_i \cap \bar{S} | = 1$, game continues, Referee reveals unique elements in \cap
3. otherwise continues

In the case (i), $M_i = T_i^1$.

In the case (ii), when the elements in \bar{S} are transmitting, the elements in S can also be transmitting and therefore the source need not receive in somecases. However some messages might go through.

Lemma 2 :

If there is an abstract broadcast protocol that terminates within K time slots, then there is a $2K$ -move winning strategy of the n_{th} hitting games.

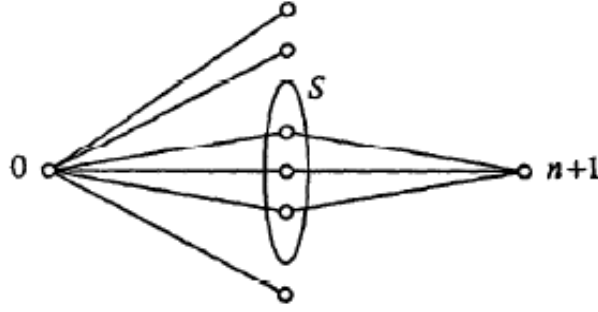


Figure 1: Three Layers consists of the last node $n + 1$, which is adjacent to nodes in S

2.1 A linear lower bound for n_{th} Hitting game

The adversary constructs a set S , that fools a sequence of M_1, M_2, \dots, M_t of moves. The problem can be defined in terms of an adaptive choice of moves. However the lower bound proof is only for a sequence of moves which are oblivious. Perhaps certain moves still can beat the adversary.

We also construct a set S for $M_1, M_2, \dots,$

starting with $S = \{1, 2, \dots, n\}$, while $\exists i, x$, such that $M_i \cap S = \{x\}$

Choose such i, x and remove x from S while there exists $\exists j, |M_i \cap S| = |M_j| - 1$

Further do remove any element of $M_j \cap S$ from S .

Lemma 3 :

The resulting S has the right intersection properties, i.e; for every i ,

1. $M_i \cap S$ is not a singleton
2. if $M_i \cap \bar{S}$ is a singleton then M_i is itself a singleton.

Lemma 4 :

If $t \leq n/2$, then output S is non empty.

Each M_i makes referee remove atmost 2 elements from S .

Given M_1, M_2, \dots, M_t , for $t \leq n/2$.

Construct S , thus game can't finish is one of the goal.

2.2 Absolute Reduction

Given M_1, M_2, \dots, M_t ; Thus n_{th} hitting game is polynomial times reducable to abstract broadcast protocol (ABP).

$$n_{th}game \leq_p ABP$$

Suppose A is an ABP, T_i^0, T_i^1, \dots, π .

$$M_{2i-1} = T_i^1 \iff \text{if } |M_{2i-1} \cap S| = 1, \text{ then } |T_i^1 \cap S| = 1$$

$$M_{2i} = T_i^0 \iff \text{if } |M_i \cap \bar{S}| = 1, \text{ then } (T_i^1 \cap S) \cup (T_i^0 \cap \bar{S}) = 1$$

With all the above results, when we run the moves from the beginning with this set S , we obtain no winning moves, which results in non ending game by the time t . Since the sink is assumed to know the ID's of their neighbors, the items $\in S$, it cannot broadcast the set S . Resulting only the smallest elements can transmit the source message; Hence in C_n graph the lower bound proof get violated.

3 conclusion

The paper shows to resolve the conflicts arising in broadcast protocols quickly by using randomization. The exponential gap between the deterministic and randomized complexities is believed to be another strong indication to the importance of randomization for distributed applications. It is reasonable to assume that processor can detect between collision and nothing. This paper claims to achieve optimal behavior in teh context of collision detection. Perhaps, though they proved lower bound it doesn't hold no more in C_n graphs.