Com S 611	Spring Semester 2007
Discrete Algorithms for Mobile and Wireless Networks	
Lecture 14 Part A: Tuesday, 6th March 2007	
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1 Lecture topic

This lecture is about the presentation of the paper "On the Time-Complexity of Broadcast in Multi-Hop Radio Networks: An Exponential Gap Between Determinism and Randomization" written by Reuven Bar-Yehuda, Oded Goldreich, and Alon Itai.

2 Models

The two models that will be discussed in this lecture are as follows. In the first model(which we will refer as $Model\ A$), if a collision occurs then no message is received by any of the stations. In the second model(which we will refer as $Model\ B$), if a collision occurs then either no message is received by any of the stations or one of the messages sent at that particular timeslot is received. Hence, no message received in a particular time slot in $Model\ A$ implies either no message was transmitted or a collision occured at the particular timeslot. Similarly, in $Model\ B$, if a message sent by some processor p_i is received at a particular timeslot, then either p_i alone transmitted a message or more stations sent messages at that timeslot. Thus, $Model\ B$ is the weaker model and more generic one, which will be tougher to work for an algorithm whereas easier to get a lower bound.

The model used in this paper is as follows. All nodes are connected to a source node (*node* 0) and only a set S of nodes are connected to the sink node (*node* $(n+1)$) as shown in Figure **??** and there are no connections between the nodes 1, 2,...n.

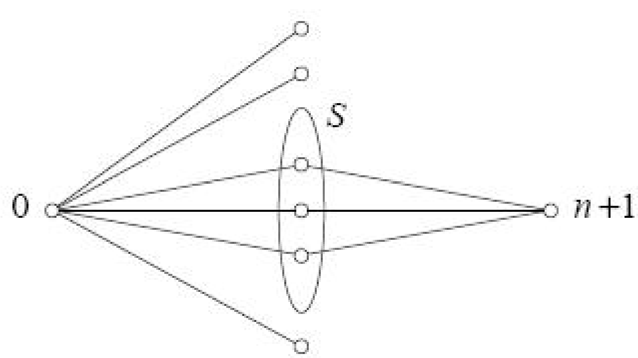

Figure 1: Model used in the Bar-Yehuda et al 1992 paper

Source node does a network broadcast which is received at all the nodes 1,2,...n. The problem here is to identify how long it takes to get a message to the sink node from the source node, when an adversary works to make this slow.

3 A deterministic lower bound

The paper describes the following protocols and comes up with reductions from one protocol to the other in the given order to come up with the lower bound.

1. General Broadcast Protocol (GBP)
2. Abstract Broadcast Protocol (ABP)
3. Restricted Broadcast Protocol (RBP)
4. Hitting Game (HG)

A reduction will be done from GBP to ABP, ABP to RBP and RBP to HG and thereby prove that if a better algorithm exists for GBP, then it can be used to solve the problem of ABP, and using this, we can solve RBP and HG, as described in the Figure **??**

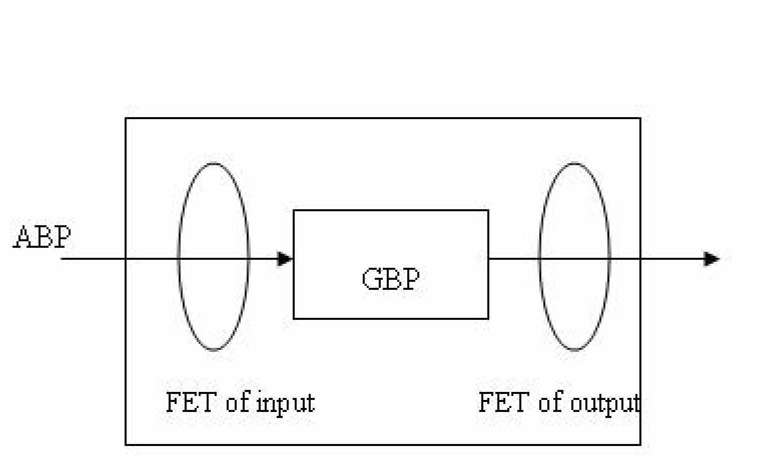

Figure 2: Reduction from ABP to GBP

We are working on a very easy network here and come up with a lower bound in this network and then prove that if we cannot come up with a better solution for this simple network, we cannot for any other complex network as well. A trivial lower bound is one time slot for a broadcast of a single message to just one host.

The paper says of reduction *to* ABP or *to* RBP and so on, whereas the actual usage should have been *from* ABP or *from* RBP and so on respectively.

3.1 Reduction

The reduction is done finally to the combinatorial game of Hitting game. The game is won by any algorithm if we are able to successfully find one timeslot in which exactly one of the nodes in S transmits. The nodes in the set S are not known If the set S is known, a simple deterministic algorithm like the the node with minimum index transmits the message to the sink, works.

The reduction is in three stages and the protocols are described as follows.

1. *GBP:* There are no restrictions in this protocol. At any time slot, a node can be either a sender or a receiver or not active. A node is considered *active* if it is either sending or receiving.

2. *RBP:* In every timeslot, exactly one of the source or the sink is active.

3. *ABP:*

    - In each time slot, only nodes in the set {1,2,...n} are transmitters and either the sink or source (not both) are receivers.
    - At the end of each slot, all nodes in {1,2,...n} know whether some node has successfully transmitted and knows the contents of the transmitted message. This assumption can be true based on some oracle algorithm which provides these nodes with the information. The assumption makes it tougher to come up with a lower bound and if a lower bound holds for a network model with this assumption, it holds for any particular model also. Note that *not* just nodes in the set S get to know of this information. Also, the assumption also *does not* mean that the set S equals {1,2,...n}.
    - The broadcast is complete once some node in S manages to transmit alone in a particular timeslot.

Lemma 1 *If there is a* RBP *that terminates in k slots on every network of the model described, then there is an* ABP *that terminates in k slots.*

The lemma is a simpler one and not proved in the class.

The description of the Hitting Game and the proof is continued in the lecture. Nalin Subramaniam has scribed the rest of the lecture.