

## 1 Papers for Reading

Li, L., Halpern, J., Bahl, V., Wang, M., and Wattenhofer, R. Analysis of a cone-based distributed topology control algorithm for wireless multi-hop networks. PODC 2001: Proceedings of the Twentieth ACM Symposium on Principles of Distributed Computing, Newport, Rhode Island, pp:264-273, August 2001

Bahramgiri, M., Hajiaghayi, M. T., and Mirrokni, V. S. Fault-tolerant and Three-Dimensional distributed topology control algorithms in wireless multi-hop networks. Proceedings of the 11th IEEE International Conference on Computer Communications and Networks (IC3N), Miami, Florida, pages 392-398, October 2002

## 2 Introduction

In the wireless network, the limitations are energy, communication and several other known issues. Perhaps, the major challenge is to save energy and reduce the possible interference in the wireless medium. Since the nodes are in wireless range, single node transmitting message can be heard by all other nodes in the neighborhood. It is high likely possible to have greater interference. The greater connectivity improves the communication of the network, however it also increases the interference. The communication connectivity can also be maintained through other nodes, reducing the interference.

The basic idea of the paper is to reduce the connectivity and reduce the interference. Perhaps, the shorter path between two nodes might become longer; however the guarantee of the path is assured. The idea of removal of unnecessary links while maintaining the required redundancy for fault tolerance and not to cause congestion and delays.

## 3 Model and Assumptions

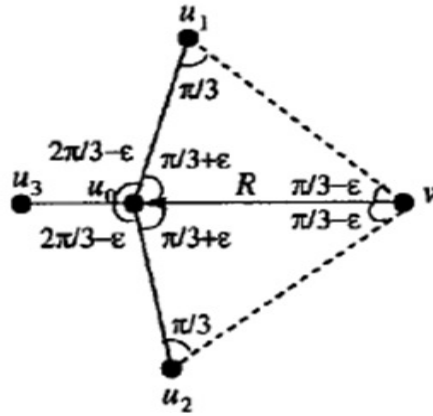
It is assumed that all the nodes in the transmission range of other node receive the message; Each node can detect the direction of arrival of the message. The nodes can modify their transmission range by varying the transmission power (maximum of  $P$ , which can reach a distance of  $R$ ). Also, the receiver node knows the senders transmission power. No nodes are equipped with location information devices.

## 4 Basic Idea

The basic idea of the paper is first to scan sliced cone, each of angle  $\alpha$  in the  $360^\circ$  two dimensional range of any node for atleast one neighbor. The goal is that if there exists atleast one neighbor in each  $\alpha$  cone then it is possible to maintain the connectivity with any other node in the network directly or through multi hop. If several neighbors exist, then the closest one, without disconnection is chosen as the neighbor and the transmission range in that angle is reduced to limited power. Thus elimination of redundant neighbors, and maintaining only required reduces the interference still maintaining the conectivity of the graph. In this process, the selection of  $\alpha$  is an important role. How much is sufficient and how much is necessary is a challenging task to obtain.

The initial version of the paper was proposed earlier which obtains that  $\alpha = 2\pi/3$  is sufficient condition. This paper tries to extend tight upper bound for  $\alpha$ . Let  $G$  be the given graph, and  $G_\alpha$  be the sugraph generated by the node removal. The neighborhood of any node  $u$  in  $G_\alpha$  is defined by  $N_\alpha(u)$ .

### 4.1 Asymmetric Edges



Let  $E_\alpha$  be the set of edges in  $G_\alpha$ . This can be formally written as below,

$$E_\alpha = \{(u, v) \in V \times V : v \text{ is a discovered neighbor by node } u\}$$

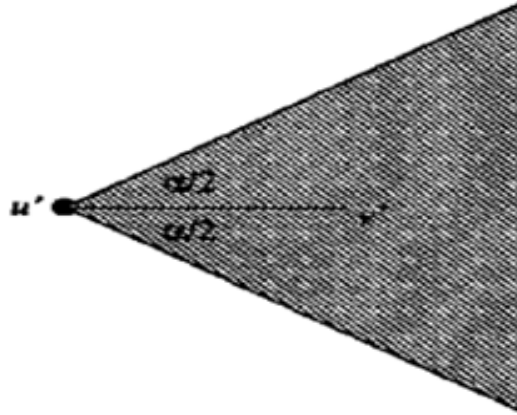
for,  $G_\alpha = (V, E_\alpha)$

Let us assume that if  $\alpha > 2\pi/3$ , say  $\alpha = 2\pi/3 + 2\epsilon$ . Then the  $\alpha$  formed may not maintain symmetric edges. For instance, if there exists a node  $u_1$  closer to  $u$  than a node  $v$  at a distance  $R$  in a cone  $u_1$  may be chosen as neighbor to  $v$ . However, if there are no other nodes other than  $u$  in the

radius of node  $v$ , then node  $u$  is chosen as a neighbor of  $v$ . Here  $E_\alpha$  need not maintain symmetry.

Simply,  $(v, u) \in E_\alpha$ , but  $(u, v) \notin E_\alpha$  means asymmetric edges.

## 4.2 Two symmetric Sets



If we consider an extension case of asymmetric edges, where all the edges are symmetric then such an edge and graph can be represented as below,

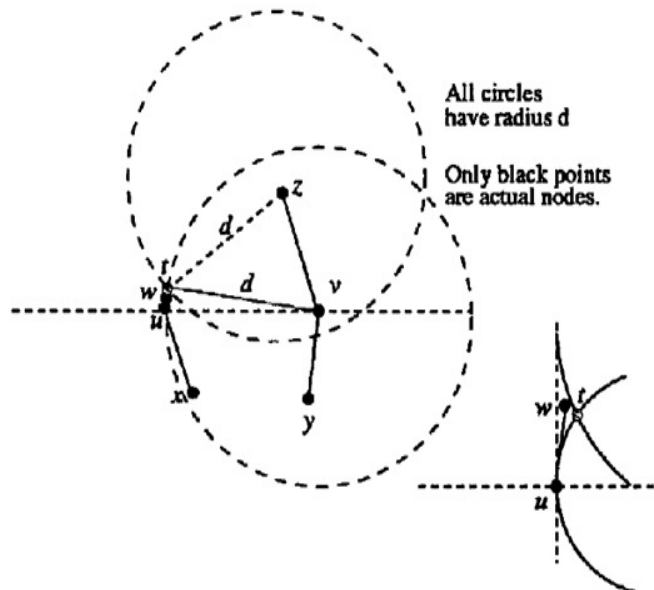
$$E_\alpha^+ = \{(u, v) : (u, v) \in E_\alpha \text{ or } (v, u) \in E_\alpha\}$$

The above one is called Symmetric closure of  $E_\alpha$ . Also, we can rewrite the equivalent  $G_\alpha$  graph as below,

$$G_\alpha^+ = (V, E_\alpha^+)$$

The figure shows such an example.

### 4.3 Connectivity Lemma



One of the important proof the paper extends is that, for  $\alpha \leq 5\pi/6$ , if  $d(A,B) = d \leq R$  and  $(A,B) \notin E_\alpha^+$ , there must be a pair of nodes, with the distance less than  $d(A,B)$ .

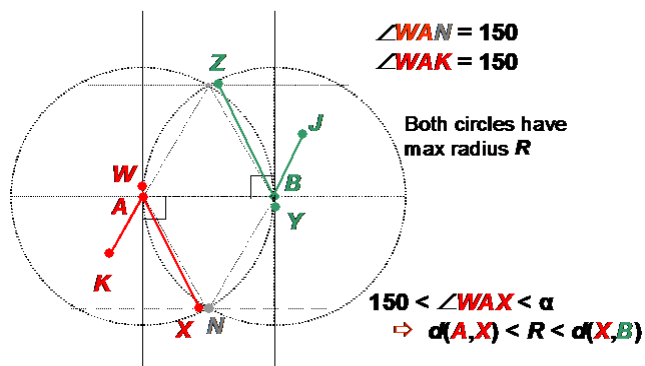
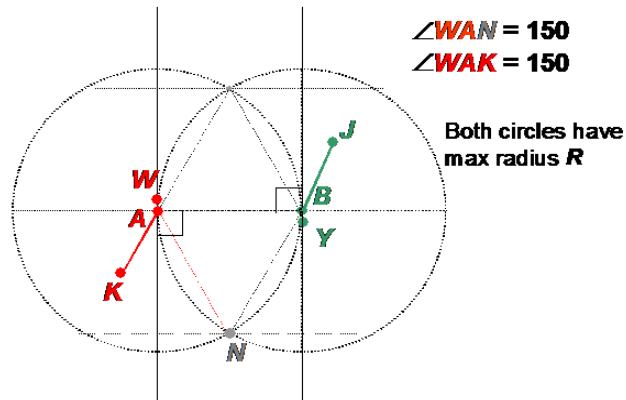
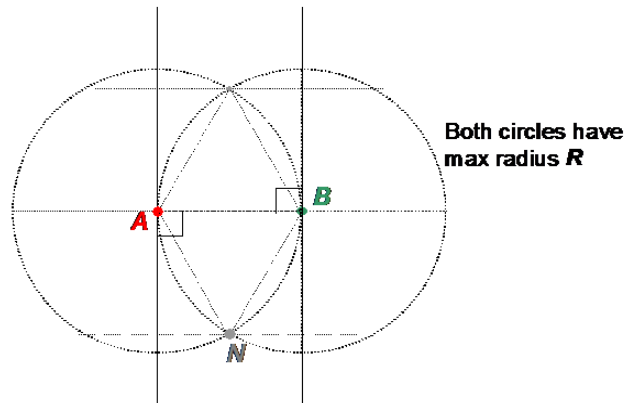
As the figure shows a perfect scenario; the proof extends that for node  $u$  there should be two nodes  $w$  and  $x$  such that the  $\angle xuw \leq 5\pi/6$ . Also for  $v$  there should be two nodes  $z$  and  $y$  such that the  $\angle zvy \leq 5\pi/6$ . This lemma forms the base for the complete paper. This assures that when  $\alpha \leq 5\pi/6$ , due to the increase in angle the requirement of the number of nodes reduces to one, there happens disconnection. However, there exists a path from node  $u$  to  $v$  via other intermediate nodes which are in the neighborhood of each other nodes respectively. That is node  $w$  and node  $z$  will always be in the neighborhood to each other or there exists a path between them. This assures the connectivity from  $u$  to  $v$ .

### 4.4 Connectivity Theorem

Ordering the edges in  $E_R$  by length and induction on the rank in the ordering, for every edge in  $E_R$ , there is a corresponding path in  $G_\alpha^+$ . Perhaps, if  $\alpha \leq 150(5\pi/6)$ , then  $G_\alpha^+$  preserves the connectivity of  $G_R$  and the bound is tight.

As stated above, the theorem is a further extension of connectivity lemma which can easily be proved by induction.

## 4.5 Necessary Condition to preserve Connectivity



Let us consider a scenario if we expand  $\alpha = 5\pi/6 + \epsilon$  further. This example shown in the figure explains that the connectivity is not preserved. Looking to the example, if A and B are

separated by a distance of  $R$ , provided the initial condition they both are neighbors. Even when two nodes  $w, k$  and  $j, y$  appears as neighbor of  $Z$  and  $B$  respectively,  $A$  and  $B$  are still neighbors.

In case if there exists a node  $x$  in the same parallel line as of intersecting point  $N$  closer to  $A$  than  $B$ , then  $(A, B)$  is eliminated. Similarly if there exists a node  $z$  in the same parallel line as of intersecting point opposite to  $N$  closer to  $B$  than  $A$ , then  $(B, A)$  is eliminated. Perhaps  $A$  and  $B$  are disconnected completely. Hence forth the connectivity is not preserved.

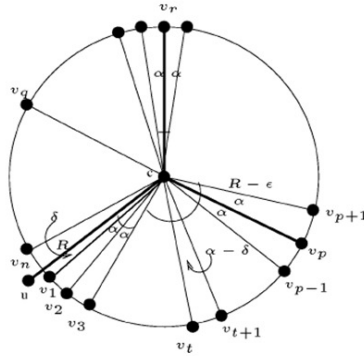
Therefore,  $\alpha \leq 5\pi/6$  is a tight upper bound for preserving connectivity in the graph.

## 5 K-Connectivity

In the above case, in each cone it is expected to assure atleast one neighbor node. However, since the nodes are viable for failure, even one node failure may cause disconnection in the network. To increase the robustness and give a fault tolerant network maintaining  $k$ -connectivity is required.

This means that each cone should have atleast  $k$ -node to maintain  $k$ -connectivity. This can be easily achieved by a simple modification to the above algorithm. Dividing each cone of angle  $\alpha$  into  $k$ -cones, with each of angle  $\alpha/k$  and running the same algorithm results in  $k$ -connectivity of the original  $\alpha$  cone. The following theorem proves the connectivity of the resulting  $G_{\alpha/k}$  graph.

Theorem: If a graph  $G$  is  $k$ -connected, then  $G_{2\pi/3k}$  is also  $k$ -connected. It means that if the CBTC( $\alpha$ ) is applied with  $\alpha = 3\pi/3k$  for a  $k$ -connected graph, then the resulting graph  $G_{2\pi/3k}$  is also  $k$ -connected. Proof: We can prove this by contradiction.



(bb)

The counterexample for  $\alpha = \frac{2\pi}{3(k-1)} + \delta$

The sufficient and necessary condition to preserve the connectivity is given by the following theorem.

Theorem: For odd  $k=2s+1$ , there exists a  $k$ -connected graph on which if we run  $CBTC(\alpha)$  algorithm with  $\alpha 2\pi/3(k-1)$  the resulting graph is not connected. In other words, it is necessary and sufficient to have  $\alpha \leq 2\pi/3(k-1)$  to preserve  $k$ -connectivity in  $CBTC(\alpha)$  algorithm.

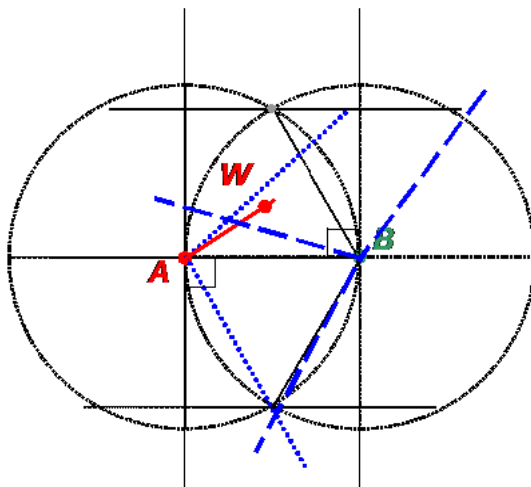
Theorem: For even  $k=2s$ , there exists a  $k$ -connected graph on which if we run  $CBTC(\alpha)$  algorithm with  $\alpha 2\pi/3k$  the resulting graph is not connected. In other words, it is necessary and sufficient to have  $\alpha \leq 2\pi/3k$  to preserve  $k$ -connectivity in  $CBTC(\alpha)$  algorithm.

## 6 Optimizations

Apart from the above algorithm, it can be further optimized by means of

1. Shrink Back operation: It is possible that there may exist an  $\alpha$ -gap, such that the algorithm keeps extending further and further the transmission power to achieve the goal. Perhaps it may end up with nothing, when it reaches to maximum power. At that point it remains as the full power utilization. This can be optimized by back tracking the boundary nodes and reducing the power level for transmission. This way optimization can be achieved.
2. In case where,  $\alpha \leq 2\pi/3$ , asymmetric edges can still be removed. The algorithm applied in this paper can still be applied for this case of  $\alpha$ . There still may exist redundant edges, which can be removed still maintaining the connectivity of the graph.
3. In case, where,  $\alpha \leq 5\pi/6$ , pair-wise edge removal can be performed. We can still perform removal of redundant edges to nodes that are closer in the neighborhood. Say for instance  $v_1$ , and  $v_2$  are in the neighborhood of  $u$  and both of them are closer to each other; then the edge  $(u, v_1)$  is a redundant of  $(u, v_2)$  or vice-versa. Hence removal of one of them still preserve connectivity of the graph.

## 7 Asymmetric Edge Lemma



For  $\alpha \leq 120(2\pi/3)$ , if  $d(A,B) \leq R$  and  $(A,B) \notin E_\alpha$ , there must be a pair of nodes, W or X and node B, with distance less than  $d(A,B)$ . The figure depicts the lemma.

Let A, B, W are nodes in the same neighbor circle radius R. As shown in the figure, if the edge (A,W) is included and (A,B) is removed if the cone contains both W, B of A. Perhaps, it is still possible that for Node B, A and W can occur in two different cones, resulting in addition of edges (B,A) and (B,W). This is perhaps redundancy.

## 7.1 Asymmetric Edge Removal

$$E_\alpha^- = \{(u, v) : (u, v) \in E_\alpha \text{ and } (v, u) \in E_\alpha\}$$

Remove rest of the edges either,

$$(u, v) \in E_\alpha \text{ and } (v, u) \notin E_\alpha \text{ or } (u, v) \notin E_\alpha \text{ and } (v, u) \in E_\alpha$$

for  $G_\alpha^- = (V, E_\alpha^-)$

## 7.2 Asymmetric Edge Removal Theorem

Two-step inductions on  $E_R$  and then on  $E_\alpha$  For every edge in  $E_R$ , if it becomes an asymmetric edge in  $G_\alpha^-$ , then there is a corresponding path consisting of only symmetric edges.

If  $\alpha \leq 2p/3$ , then  $G_\alpha^-$  preserves the connectivity of  $G_R$  and the bound is tight.

Proof: This can Easily be proved by induction

## 8 3-Dimensional Extension

As in the above case we have considered every proof for 2-D space. The second paper extends the proof to 3-D space by constructing a 3D cone with the intersection sphere. The results are as follows:

For 1-connectivity,  $\alpha \leq 2\pi/3$  and

For K-connectivity,  $\alpha \leq 2\pi/3K$

to preserve the connectivity. Any angle above the results may cause disconnection of the network.

## 9 Summary

The CBTC is an algorithm which exploits the angle of arrival, and utilizes the redundant edge removal in order to reduce the interference in the network, perhaps assuring the connectivity of the graph by itself. Also they provide tighter upper bound for 2D and 3D space. Also they have extended the work for K-connectivity. The paper also suggests several optimization methodology for practical applications.