Com S 611	Spring Semester 2007
Discrete Algorithms for Mobile and Wireless Networks	
Lecture 10 Part B: Thursday, 15th February 2007	
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1 Introduction

This lecture notes is based on the presentation of the paper "Distributed Topology Control for Power Efficient Operation in Multihop Wireless Ad Hoc Networks "(Wattenhofer, Li, Bahl and Wang) by Bibudh Lahiri.

The lifetime of a wireless network that is operating on battery power is limited by the capacity of its energy source. A node in a wireless network independently explores its surrounding region and establishes connections with other neighboring nodes that are within its transmission and reception range. In establishing these local connections, it is desirable to choose only those local connections that will guarantee overall global network connectivity while satisfying different and often contradictory performance metrics such as overall throughput, network utilization, and power dissipation. Unlike wired networks, each node in a multihop wireless network can potentially change its set of one-hop neighbors and consequently the overall network topology by simply changing its transmission and receive power. Without proper topology control algorithms in place a randomly connected multihop wireless ad hoc network may suffer from poor network utilization, high end-to-end delays, and short network lifetime.

2 Related Work

Although the problem of topology control is a pretty fundamental one, there has been only a limited amount of work in this area that aimed towards increasing network longevity. The following few are worth mentioning:

- **Triangulation-based algorithm for logical links:** Hu described a distributed triangulation-based algorithm for choosing logical links following a few heuristic guidelines. But his algorithm did not take advantage of adaptive transmission power control.
- **Centralized spanning tree algorithm:** Ramanathan and Rosales-Hain described a centralized spanning tree algorithm for creating connected and bi-connected static networks, minimizing the maximum transmission power for each node. But their algorithm did not guarantee network connectivity in all cases.
- **Distributed topology control algorithm:** Meng proposed an algorithm that guaranteed connectivity of the entire network, but relied on a radio propagation model.
- **Adaptive clustering-based routing protocol:** This protocol proposed by Heinzelman et al. maximized the network lifetime by randomly rotating the role of cluster-heads among nodes with higher energy reserves.

3 Objectives

The algorithm in this paper was developed with the following objectives:

- The nodes should only use local information for determining transmission radius, and make decisions to guarantee global node connectivity.
- The nodes should minimize power consumption by finding minimum power paths.
- The nodes should find a topology with small node degree, so that interference is minimal.

- The algorithm should be simple and efficient, because the nodes have very limited computing resources and memory.
- The algorithm should not make strong assumptions about the radio propagation model or the underlying hardware.

4 Difference with previous work

The algorithm in this paper is different from the earlier work in the following ways:

- This algorithm guarantees that the maximum connected set of nodes for the network will always be found.
- This algorithm is computationally less demanding, and it does not specify a deployment region.
- This algorithm does not need exact location information but only directional information.
- This algorithm is able to offer a worst-case analysis for both the minimum power routes and the maximum node degrees in the network.

5 Model

We work with a set V of n nodes deployed on a plane. A node does not know its position. Nodes broadcast messages with power p, which can have a maximum value P. p is assumed to be unknown function of distance. The radio communication unit is able to determine the direction of the sender when receiving a message. If two nodes u and v exchange a message pair, then node u knows that node v is in direction ρ , and node v knows that node u is in direction $\rho+\pi$, with $0 \leq \rho < 2\pi$. We denote the set of neighbors of node u as $N(u)$.

6 The Cone-based Algorithm

The algorithm has two phases. The first phase is a decentralized scheme that builds a connected graph upon our node network by letting nodes find close neighbor nodes in different directions. The second phase improves the performance by eliminating non-efficient edges in the communication graph.

6.1 Phase One: The Neighbor Discovery Process

In this phase, each node u beacons with growing power p, initially $p = \epsilon$. If node u discovers a new neighbor node v, node u puts v into its local set of neighbors $N(u)$. Node u continues to grow the transmission power until the neighbor set $N(u)$ is big enough such that, for any cone with angle α there is at least one neighbor $v \in N(u)$, or until node u hits the maximum transmission power P. An important fact which was discussed in the class, and which is worth mentioning here is that, when we mention having at least one neighbor in any cone with angle α, *we do not divide the whole* $360°$ *around* u *into non-overlapping cones each with angle* α, rather, we mean *there should be at least one neighbor in the cone with angle* α*, whatever be the orientation of the cone*.

If a node u with maximum transmission power P has a cone C = [ρ, ρ + α] without any node in $N(u)$, then node u decreases its transmission power, back to the minimum power p such that all the cones that had some neighbor when transmitting with maximum power P, still continue to have some neighbor. This algorithm is symmetric, that is, if node u wants node v to be in its neighbor-set, then node v also needs to put node u in its neighbor-set.

From the algorithm description of phase 1 we conclude:

Fact 1 *For each node* u *and for each angle* ρ *(*$0 \leq \rho < 2\pi$*), if there is a node* v *in the cone* C *= [*ρ*,* ρ *+* α*] when sending with maximum power* P *, then there is a node* v' *in the cone* C *when sending with minimum power* $p(u)$*.*

6.2 Phase Two: The Redundant Edge Removal Process

Once a connected graph is formed in Phase 1 of the algorithm, we remove all edges (u, w) where there exists some multi-hop path between u and w whose power consumption is lower than the direct edge, i.e., *a direct edge is kept only if it offers some advantage from the point of power consumption, over a multi-hop path.*

Formally speaking, if there are two nodes v, w with $v, w \in N(u)$, $w \in N(v)$ and $p(u, v) + p(v, w) \leq p(u, w)$, then we remove node w from $N(u)$.

From a performance point of view, a node should have as few neighbors as possible. Thus we might consider removing nodes from our neighborhood *even though a direct transmission uses less power than an indirect*. Formally, if there are two nodes v, w with $v, w \in N(u)$, $w \in N(v)$, $p(u, v) \leq p(u, w)$ and $p(u, v) + p(v, w) \leq q * p(u, w)$, then we remove node w from $N(u)$ (and by symmetry also u from $N(w)$).

Fact 2 *For each node u, if there was a neighbor node $w \in N(u)$ after the first phase of the algorithm, there is a neighbor $v \in N(u)$ after the second phase of the algorithm such that $p(u, v) + p(v, w) \leq q * p(u, w)$, for a constant $q \geq 1$.*

Definition 1 *A path p of nodes is an ordered set $(u_1, u_2, \ldots, u_k)$ of nodes such that there is an edge between consecutive nodes: $e = (u_i, u_{i+1})$ for $i = 1, 2, \ldots, k-1$ with $e \in E$.*

Definition 2 *A graph is connected if there is a path from any node to any other node in the graph.*

Definition 3 *The distance of two nodes is their Euclidean distance in the plane.*

Theorem 1 *Let $G = (V, E)$ be the graph constructed by the algorithm discussed in the paper. Let $G' = (V, E')$ be the connection graph when all nodes always beacon with maximum power P. If $\alpha \leq 2\pi/3$, then G will be connected if G' is connected.*

Proof: We prove it by contradiction. Suppose G' is connected but G is not. Then there exists at least a pair of nodes such that there is no path between the pair. Let the nodes u, v be the pair with minimum power to beacon each other, that is $p(u, v) \leq p(u', v')$ for any pair of nodes u', v' without a path. Since G' is connected we know that $p(u, v) \leq P$.

Let $d = p^{-1}(u, v)$, i.e., with power $p(u, v)$ one can reach distance d. The algorithm has given node u minimum transmission power $p(u)$. Since there is no edge $e = (u, v)$, the maximum distance one can reach with $p(u)$ is $p^{-1}(u) < d$.

The rest of the proof follows from some geometric properties of triangles. Let w be a neighbor node of u. We construct a triangle with the nodes u, v and w, as shown in Figure 1.

From cosine rule, we know $c^2 = a^2 + b^2 - 2ab\cos\gamma$. We have the following parameters:

$a = d(u, v) = p^{-1}(u, v) = d$
$b = d(u, w) = p^{-1}(u, w) \leq p^{-1}(u) < d$
$c = d(v, w) = p^{-1}(v, w) \geq p^{-1}(u, v) \geq d$

It follows that $a > b$ and $c > b$

Hence $\cos\gamma = \frac{a^2+b^2-c^2}{2ab}$
Now, with the given parameters, $a^2 + b^2 - c^2 < b^2$
Hence $\cos\gamma \leq \frac{b^2}{2db} < \frac{1}{2}$ which implies $\gamma > \pi/3 \geq \alpha/2$. Therefore there is no node $v' \in N(u)$ in the cone $C = [-\alpha/2, \alpha/2]$, which contradicts Fact 1. By symmetry the same holds for v.

This algorithm not only results in a connected graph, but also produces routes that are very power efficient.

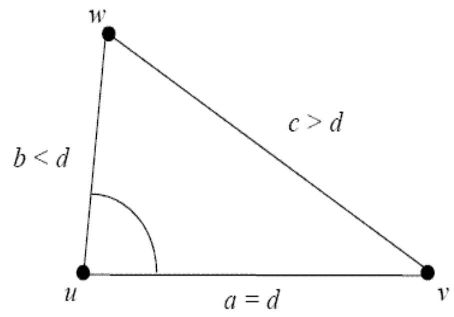

Figure 1: Triangle u,v,w

Definition 4 *The power consumption of route* r = (s=u_1,u_2,...,u_k = t) *is* $C(r) = \sum_{i=1}^{k-1} p(u_i, u_{i+1})$

Definition 5 *We are assuming a power model now. A direct transmission from* u *to* v *costs power* $p = p(u, v)$*, where* p *is a function of the distance* $d = d(u, v)$*. Any function* $p(d)$ *is eligible as long as* $cd^x \leq p(d) \leq czd^x$*, for parameters c,x,z, all independent of d, and with* $z \geq 1$*, and* $x \geq 2$*.*

Definition 6 r^* *is a minimum power route if* $C(r^*) \leq C(r)$ *for all routes r from s to t.*

Theorem 2 *Let* G *be the graph constructed by the algorithm in the paper, and* G' *be the graph when all nodes transmit with maximum power* P*. Let* s *be a source node and* t *be a sink node. Let* $C(\hat{r})$ *and* $C(r^*)$ *be the minimum power routes in* G *and* G'*. If* $\alpha \leq \pi/2$*, then* $C(\hat{r}) \leq C(r^*)zq(1 + 2\sin(\alpha/2))$*, for the radio model described in definition 5.*

The proof of this theorem was not covered in the presentation. However, theorem 2 provides an upper bound on the power consumption by any route created by the algorithm discussed in the paper.

7 Simulation Model

This topology control algorithm was implemented in ns-2, and tested on a network of 100 nodes placed uniformly at random in a rectangular region of $1500m$ by $1500m$. A $\frac{1}{d^4}$ transmit roll-off was assumed. All nodes periodically sent UDP traffic to a master data collection site situated at the boundary of the network.

8 Results

8.1 Analysis of the Resulting Topology of Different Topology Control Algorithms

The average degree of the multihop wireless networks should not be too large because a large average degree typically implies a node has to communicate with other distant nodes directly. This increases interference and collision, and would waste energy. On the other hand, the average degree should not be too small either because that tends to increase the overall network energy consumption as longer paths have to be taken.

The simulation results for this algorithm reveal that *the average node degree of the graph constructed by Phase 1 of the algorithm increases as* α *decreases* (Figure 2 and 3). The boundary nodes contribute more to the average degree statistics because in trying to cover the maximum angle, it tends to involve more distant nodes. Phase 1 of the algorithm yields a topology with a much higher average degree than one than yielded by the centralized spanning tree algorithm proposed by Ramanathan and Rosales-Hain (referred to as R&M in the figures). However, after applying Phase 2 of the algorithm, the redundant edges get removed and we get a similar low degree topology graph as the R&M algorithm (Figure 4,5,6 and 7).

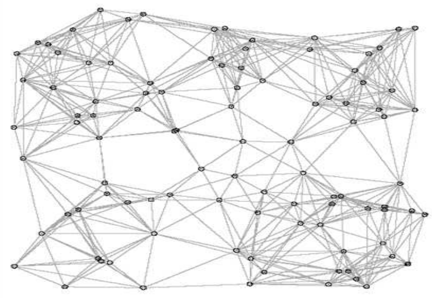

Figure 2: Topology graph for Phase 1 Only with $\alpha = 2\pi/3$

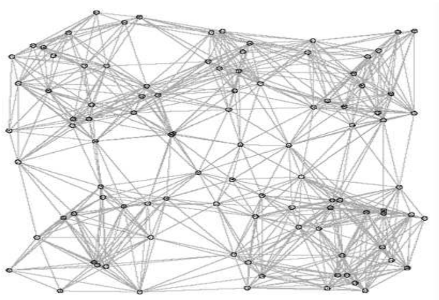

Figure 3: Topology graph for Phase 1 Only with $\alpha = \pi/2$

8.2 Network Performance Analysis of Different Topology Control Algorithms

Network lifetime in the multihop wireless networks environment is a matter of much concern. The network lifetime is measured as the number of nodes still alive over time.

The ConeBased algorithm proposed in this paper performs as good as the R&M algorithm, while using only directional information. They both perform significantly better than MaxPower. From Figure 8, we see that when 80% of the MaxPower nodes are dead, both ConeBased and R&M still have around 90% of nodes alive. If only Phase1 of the algorithm is applied without the edge-removal optimization, it does not perform as good as the ConeBased algorithm or the R&M algorithm, but it performs much better than no topology control case. When 80% of the MaxPower nodes are dead, Phase1Only still has more than 60% of nodes alive. A constant number of nodes stay alive for all the topology control algorithms except MaxPower.

Figure 9 shows how the network topology evolves over time. The topology control algorithms tend to maintain the same average node degree for the remaining alive nodes as nodes die over time. The average node degree decreases noticeably only when the network has less than 40% nodes alive. Since MaxPower do not respond to topological changes, the average node degree decreases quickly over time.

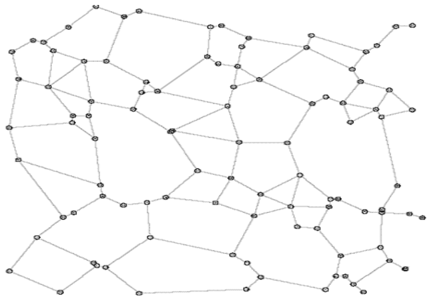

Figure 4: Topology graph for Cone Based with $\alpha = 2\pi/3$

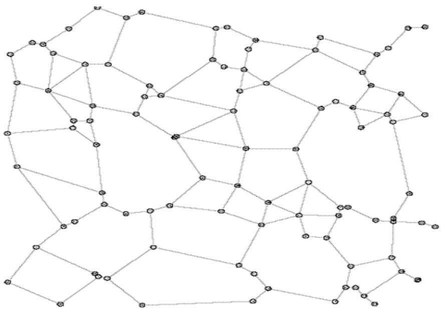

Figure 5: Topology graph for Cone Based with $\alpha = \pi/2$

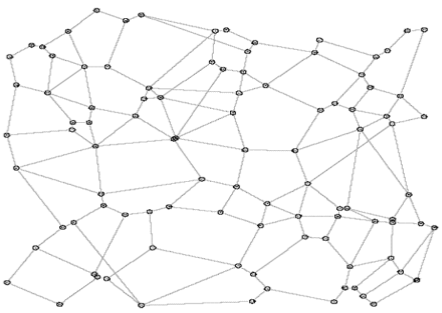

Figure 6: Topology graph for R&M

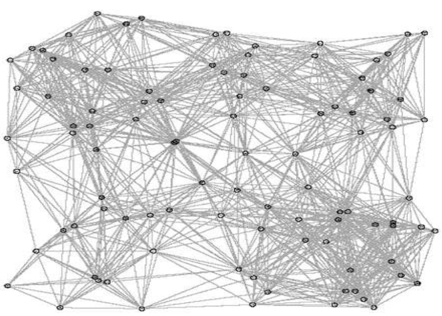

Figure 7: Topology graph for Max Power

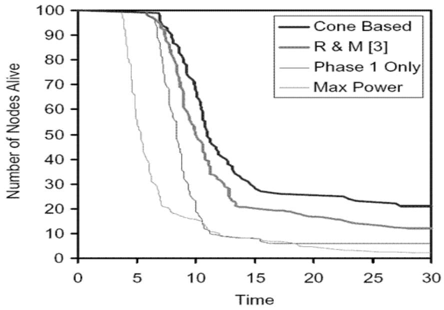

Figure 8: Network lifetime

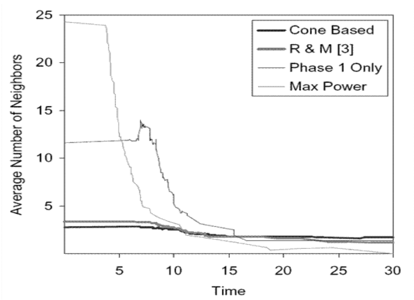

Figure 9: Average node degree over time