Computational Modeling of Deformation and Impact

This thesis tackles several problems arising in robotics and mechanics: robotic pickup of soft three-dimensional objects, recovery of their gravity-free shapes, analysis and computation of two- and multi-body impacts, and solution of the problem of an elastic rod hitting ground with viscous damping.

Impact is an event that lasts a very short period of time but generates a very large interaction force. Assuming Stronge's energy-based restitution, a formal impulse-based analysis is presented for the collision of two rigid bodies at single contact point under Coulomb friction in three dimensions (3D). Based on this analysis, a complete algorithm is described to take advantage of fast numerical integration and some closed form formulas. Some related strategies for batting a flying object into a target in 3D are proposed.

Multi-body collision often occurs when several objects collide together, as observed in the games of marbles, billiards, and bowling. A general computational model is proposed for predicting the outcome of an n-body collision.

An alternative way of modeling impact is to consider the engaged objects to be elastic rather than rigid. A damped one-dimensional wave equation can model an elastic rod bouncing off the ground at a given initial velocity, under the influence of gravity. Based on the Method of Descent and D'Alembert's formula, an explicit solution is derived. The time of contact with the ground is analyzed and determined in terms of the initial velocity, damping coefficient, and gravitational constant.